multille m # " multille m # " multille # # " Institute # ** * mating m # 3 matinu m # 3 NC High School Mathematics Contest Finals 2018: Level I Ro tinstitute \$ Find the number of integers that are in the region defined by the inequality $5 < |n| \le 73$. (A)136 (B) 134 (C) 68 (D) 67 (E) None of these 2. Let n, n + 1 be positive intergers such that their quotient is 1.02. Compute the product of n and n + 1. 🕥 (E) None of these (A)101 (B) 202 (C) 2550 (D) 5250 Y. with domain $D = \{x | x \neq 0, x \neq 2\}$. Evaluate 3. Consider the function $f(x) = \frac{x^6 + 3x^5 - 4x^4 - 12x^3}{x^4 - 2x^3}$ f(9997). 面对加根教林省際 (B) 99997000 (C) 9999900 (D) 99990000 (E) None of these (A)9997000 Ro 加海林 Institute 4. How many real solutions are there to the equation |200x + |1000x - 800|| = 400?Withit the the the the Y. (C) 2 (E) None of these (A)0 (D) 4 (B) 1 സ institute # Suppose a point is selected at random inside a circle. Find the probability that the point is closer 5. to the center of the circle than the edge. Institute # # '\$ Y. (C) $\frac{3}{4}$ (D) $\frac{5}{8}$ (E) $\frac{7}{8}$ $(B)\frac{1}{2}$ 6. In an isosceles triangle ABC, $m \angle C = 30^{\circ}$ and AC = BC. Suppose AC = a. Find BD if D is on \overline{BC} such that \overline{AD} is the altitude of triangle ABC. Artitute the the 's the N. (A) $\frac{(3-\sqrt{5})a}{2}$ (B) $\frac{(\sqrt{5}-1)a}{2}$ (C) $\frac{(\sqrt{3}-1)a}{2}$ (D) $\frac{(2-\sqrt{3})a}{2}$ (E) $\frac{1}{3}a$ 面的机能称林塔张 Astitute # # " PR Astitute # # '% PK stitute # # 3 PS Autitute the tot is the Withthe the the the Y. to the the B to the the By the to the We By M 10 the H- 13 % Y. the the the 's to the the 's

multille m # * multinu m X 3 Mustitute 300 X Institute \$7 % multilite the the second Institute \$7 \$7 'S 7. Consider the sequence $\{a_n\}$ that is defined recursively by Y. $a_1 = 10$, $a_{n+1} = \sqrt{a_n}$ for $n \ge 1$. If the sequence is continued indefinitely, which of the following the sequence is continued. following numbers gives the limit of $\{a_n\}$? $(A) 2\sqrt{5}$ (B) $\sqrt{21}$ (C) 3 (D) 5 (E) 1 8. Three athletes run a 1500 meter- race. When the first runner finishes, the second runner and third runner are at 100 and 170 meters from the finish line, respectively. If each runner maintains a constant speed, how far is the third runner from the finish line when the second runner finishes? Withte the the the file 而时间他新林等除 (A)70m(C) 74.5 m «С (E) 77 m (B) 72.5 m (D) 75 mY. finistitute An 9. Find the area of the regions enclosed by the equations: |x| - |y| = 1 and |x| = 3. Ro (A) $12 unit^2$ (B) $8 unit^2$ (D) $5 unit^{2}$ (E) $3 unit^{2}$ (C) $6 unit^2$ institute ## 10. A virulent infectious disease spreads as given by the equation $N(t) = N_0(1 - 10^{-kt})$ where: t is time in hours, matilite # # 3 PS N(t) is number of people infected at time t Ro N_0 is the total population of the community at time t = 0k is the infection rate constant of the disease (k = 0.001). Calculate how long will it take for 90% of the population to be infected. (A)90 hours(B) 100 hours (C) 1000 hours (D) 990 hours (E) None of these ANTIMA AT # 18 18 $\frac{4x+9}{5} - \frac{2x+3}{3} = \frac{x-5}{2}$ N. 11. Let *m* be the solution of the equation Compute the value of $m^2 + m$. astitute ###### (E) 90 新光·塔化 *3 Ph N. (D) 85 (A)61 (B) 72 (C) 80 而如此他就林塔张 Astitute the the " free Astitute # # 13 PR Withte # # 3 PR Willith # # 13 PR Withthe the the the Y. to the the B to the the By the to the We B The

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Institute \$ 75. 3 multille m # 3 Institute # ** multille # * * multille m # 3 Institute # # 3 matitute # # 3 PE hitte the the the the $\left(\frac{1}{1+\sqrt[3]{2}+\sqrt[3]{4}}\right)^{3}$ 12. Simplify the expression: Y. tinstitute ## # (B) $1 - \sqrt[3]{2} - \sqrt[3]{4}$ (C) $1 - 3\sqrt[3]{2} + 3\sqrt[3]{4}$ (D) $\sqrt[3]{2} - 1$ $1 + 3\sqrt[3]{2} - 3\sqrt[3]{4}$ (A) 前期的新林塔梯 (E) None of these (E) None of these 13. Let p be the solution of the equation $5^{2x-1} \cdot 10^{6x} = 8^{2x}$. Find the value of $(\sqrt[3]{p} + p)^2$. 化物林 塔林 N. (A) 2 (B) $\frac{2}{3}$ (C) $\frac{1}{64}$ (D) $\frac{25}{64}$ (E) None of these 14. Consider the sequence { a_n } such that $a_0 = 4$, $a_1 = 2$, $a_n = a_{n-1} - a_{n-2}$. Compute the sum: Y. multinte ## # $a_0 + a_1 + a_2 + \dots + a_{101}$. (C) -2 (D) -4 (E) 0 (A) + (B) 2 (C) -2 (D) -4 (E) 0 15. Consider the real function $f(x) = ax^3 - bx - 5$ such that f(1) = 0. Find f(-1). R myittle # # (D) –10 (E) -20 (A)1 (B) - 1(C)-5Y. institute # 16. Suppose the quadratic equation $x^2 + kx + 2k = 0$ has at least one real solution. Find the criteria satisfied by k. (C) $k \leq -1$ or $k \geq 9$ (D) $k \leq 0$ or $k \geq 9$ (E) $k \leq 0$ or $k \geq 8$ $(A)k \le -1 \quad (B)k \ge 9$ Y. 17. Suppose the function $f(x) = x^2 + 2bx + b$ has two x-intercepts such that one of the x intercepts has a value of -3. Compute the value of the other x-intercept. Withit the Her Her & Ph $(C) - \frac{5}{3}$ (D) $- \frac{3}{5}$ (E) $\frac{6}{5}$ (B) **–**2 No. 18. A triangle has sides of lengths 4, 5, and 6. Find the length of the altitude to the side of length 6. $(A)\frac{5\sqrt{7}}{3}$ institute ## # '\$ 1% (C) $\frac{5\sqrt{7}}{4}$ (D) $\frac{24}{5}$ (E) $\frac{5\sqrt{7}}{6}$ N. to the the B Ph to the the 'S Ph to the the B to the We the to the We B We to the W. B. Ro

19. Consider the triangle *ABC*, with vertices with coordinates;
$$A = (4, -3)$$
, $B = (4, 1)$, and $C = (2, 1)$. Calculate the area of the triangle *ABC*.
(A) $\frac{1}{8}$ (B) $\frac{1}{4}$ (C) $\frac{1}{2}$ (D) 4 (E) None of these
20. Consider the function $f(x) = x^2$. Suppose the following sequence of transformations are performed on $f(x)$ to obtain $g(x)$.
1. Translation of 2 units to the left.
1. Vertical shift of 3 units up.
13. Vertical shift of 3 units up.
14. Vertical shift of 3 units up.
15. Vertical shift of 3 units up.
16. Vertical shift of 3 units up.
17. Vertical shift of 3 units up.
18. Vertical shift of 3 units up.
19. Translation of 2 units to the left.
19. Vertical shift of 3 units up.
19. Vertical shift of 3 units up.
10. Vertical shift of 3 units up.
11. Vertical shift of 3 units up.
12. A) -5 (B) -3 (C) 3 (D) 5 (E) 13
21. Let x and y be real variables such that $x \ge 0$ and $y \ge 0$. Suppose
13. $f(x, y) = (x^2 + y^2 - 2)^2 + 4(xy + 2)(x^2 + xy + y^2)$. Compute $\sqrt{f(x, y)}$.
(A) $x^2 + y^2 = (1)(x^2 + y^2 - 2)^{1/2} + 2xy (C) 2(xy + 2)^2 (D)(x + y)^2 + 2 (1)(4xy)$
22. The altitude perpendicular to the hypotenuse of a right triangle is 12 cm. Express the length of the hypotenuse h as a function of the perimeter t .
(A) $h = \frac{p^2}{2r+24}$ (B) $h = \frac{p}{2r+44}$ (C) $h = \frac{p^2+24}{27}$ (D) $h = \frac{2r+44}{2r^2}$ (E) $h = \frac{p}{2r^2+26}$
23. Twenty-four different 4-digit numbers can be arranged by re-arranging the digits of the number 1234. Find the sum of these twenty-four numbers.
(A) 57661 (B) 67542 (C) 76563 (D) 66664 (F) 66660

the variance of S be given by V(S) such that: 24. Consider the set $S = \{a_1, a_2, a_3, \dots, a_n\}$. Let the arithmetic mean of S be denoted by E(S) and itute \$ itute ##

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$$E(S) = \frac{1}{n} \sum a_i$$
$$V(S) = \frac{1}{n-1} \sum (a_i - E(S))^2$$

If each element of S is multiplied by k and then increased by l, find the value of the new variance of the set S.

(A) E(S) + kl (B) $k^2V(S)$ (C) $k^2V(S) + l$ (D) $l^2V(S) + k$ (E) $(E(S))^2 - k^2$

1/3 1/2 25. Consider the triangle PQR. Let the points A, B, and C be the midpoints of the sides PQ, QR, and *PR*, respectively. Given that the midpoint coordinates are such that A = (-4, 1), B = (-2, 2), and C = (-5,3), compute the area of triangle PQR.

(A) 40 unit^2 (B) 20 unit^2 (C) 10 unit^2 (D) 4 $unit^2$ (E) None of these the star th

institute # 26. In the mythical city of Jamais, the 6-faced die used in Casino Mathematicale has the face numbers {1, 2, 3, 4, 5, 6} handcrafted such that the probability of a number x appearing during a random toss is proportional to the number x. Compute the probability that when a (E) None of these the second s single die is tossed randomly, an odd prime number is observed. astitute ##

(D) $\frac{3}{4}$

(x,y)

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 $(A)\frac{2}{9}$ (B) $\frac{1}{7}$

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面对机机都林塔路 If the equation of L is 2x + y = 100, find the value of the maximum possible area of the shaded rectangle.

(0,0)

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Astitute # # 13 PS (A) 1150 unit^2 (B) 1250 unit^2 (C) 1300 unit^2 (D) 1350 unit^2 (E) None of these A) 林林林 thillite # 3 Astitute the the second stitute # #

Institute m # " Institute # ** * Institute # # " multille m # 3 Institute \$ ** ** multinu m # 3 stitute # # ' K 28. Consider the binary operations © and ® defined over the real numbers by the expressions $a \mathbb{C}b = 2^{a+b}$ and $a \mathbb{B}b = 2^{ab}$. Suppose $2\mathbb{B}(2\mathbb{C}2) = 2x\mathbb{C}4$. Solve for x. Y. (A)4 (B) 6 (C) 12 (D) 14 (E) 32 29. Consider the ordered triple $\{a_1, a_2, a_3\}$ such that $\frac{1}{1+\sqrt{2}+\sqrt{3}} = a_1 + a_2\sqrt{2} + a_3\sqrt{6}$. Find the sum of a_1, a_2 , and a_3 . (A) 0 (B) $\frac{1}{6}$ (C) $\frac{1}{3}$ (D) $\frac{3}{8}$ (E) $\frac{1}{2}$ Ro 30. Suppose a, b, c, and d are non-zero real numbers. If $ax^2 + bx + c = 0$ has two real solutions of opposite signs and $ax^2 + bx + c = d$ has two real solutions of the same sign, which of the following statements is true? (B) ac < 0, |c| > |d| (C) ac > 0, |c| < |d|(E) ac > 0, c > dmistille # # 3 PE (A) ac < 0, |c| < |d|Ro (D) ac > 0, |c| > |d|31. Let x be a real number. Consider the equation $x^{1/2} - 9x^{1/3} + 20x^{1/6} = 0$, $x \ge 0$. Compute the sum of the values of $x^{1/6}$. 而就加根類林婆佛 (C) 11 (D) 15 (E) 20 Y. 80 (A)9 **(B)** 10 32. Anthony's mother is 20 years older than Anthony, but she is 3 years younger than Anthony's father. Anthony's father is 7 years younger than 3 times Anthony's age. Find the sum of their ages. Y. (A) 66 years (B) 77 years (C) 88 years (D) 102 years (E) None of these withthe ## # # 18 33. Let f(x) be a real function such that $f(x) = x^{17} - x$ where x is a real number. Find the N. number of real roots of f(x). stitute \$ (A)8 (C) 3 (D) 2 (E) 1 (B)4而如此他教林後然 avitute # # * * Institute the tet 's PR Withthe the the is the Withthe \$6 # 'S PS Willing the the the Y. to the the B. Ph to the the the to the bear of the to the the B Ro the the the 's to the the 's

34. A rectangular storage container with an open top has a volume of $10 m^3$. The length of its base is twice its width. Material for the base costs \$10 per square meter; material for the sides costs \$6 per square meter. Express the cost C of material as a function of the width of the base w.

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(A)
$$C(w) = 10w^2 + \frac{180}{w}, w > 0$$
 (B) $C(w) = 20w^2 + \frac{360}{w}, w > 0$
(C) $C(w) = 20w^2 + \frac{180}{w}, w > 0$ (D) $C(w) = 10w^2 + \frac{360}{w}, w > 0$ (E) None of these

35. Compute the maximum value of P(x, y) = 4x + 5y subject to the constraints $x \ge 0$, $y \ge 0$, 2x + 2y < 10 and x + 2y < 6. $2x + 2y \le 10$, and $x + 2y \le 6$.

- (A) 19 (B) 20 (C) 21 (D) 22 (E) None of these 36. Consider the inequality $|x 2| \le 7$. Let (a_1, a_2) be an ordered pair such that $a_1 \le \frac{1}{x 10} \le a_2$. mutute ##
 - Compute (a_1, a_2) .

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(A) (-15, -1) (B) (-1, -15) (C) $\left(-1, -\frac{1}{15}\right)$ (D) $\left(-\frac{1}{15}, -1\right)$ (E) $\left(1, \frac{1}{15}\right)$

37. Determine the last digit of 2018^{2018} .

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(A)2 (B) 4 (C) 6 (D) 8 (E) None of these 38. Let $a = \frac{x}{2018} - 2018$, $b = \frac{x}{2018} - 2016$, and $c = \frac{x}{2018} - 2020$ where $x \neq 0$. Find the value of $a^2 + b^2 + c^2 - ab - bc - ca$. CA A A A

(A)4 (B) 8 (C) 12 (D) 16 (E) 24

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39. The probability of rolling two dice and observing a double-1 (making a sum of two, or "snakeeyes"), is 1 in 36. Calculate the probability of getting a double-1 on at least one of the two rolls multille # # B of a pair of unbiased dice. (A) $\frac{1}{6}$ (B) $\frac{1}{3}$ (C) $\frac{71}{1296}$ (D) $\frac{1}{1296}$ (E) $\frac{1}{18}$ itute # #

Withite # # 3 PS 40. Let $f(x) = 1 + x + x^2 + x^3 + x^4$. Find the remainder when f(x) is divided by $x - \frac{1}{10}$. (A) 10.9999 (B) 9.9999 (C) 3.3333 (D) 2.2222 (E) 1.1111 Ro
