State Mathematics Finals: Level III
April 28, 2016

1. Which of these points is on the line connecting (3, 7) and (5, 4)?

a. (4, 5) b. (–1, 13) c. (8, 11) d. (9, –1) e. none of these

2. Let $f(x) = \dfrac{2x+3}{x}$. Find the value of x where $f(x) = 8$.

a. 0.5 b. 2 c. 2.375 d. 2.625 e. 5

3. Determine the area of a rectangle with a perimeter of 28 cm and a diagonal 10 cm.

a. 48 cm^2 b. 49 cm^2 c. 50 cm^2 d. 160 cm^2 e. 180 cm^2

4. Six coins (three are gold and three are silver) are evenly distributed among three kids. If the process is random, what is the probability that each kid will have a golden and a silver coin?

a. $\frac{1}{20}$ b. $\frac{1}{6}$ c. $\frac{195}{720}$ d. $\frac{1}{3}$ e. $\frac{2}{5}$

5. Carl starts at 7 a.m. for his morning run. His pace is 10 km per hour. Margi starts a half hour later trying to catch up to him on her bike going 30 km per hour. When will she catch up?

a. 7:10 a.m. b. 7:15 a.m. c. 7:40 a.m. d. 7:45 a.m. e. 7:54 a.m.

6. Arrange p, q and r order of size from smallest to largest, where:
$p + q + r = 10$, $p + 2q + 3r = 22$ and $p - 4q + 9r = 31$

a. $p < q < r$ b. $p < r < q$ c. $q < r < p$ d. $q < p < r$ e. $r < q < p$

7. How many distinct real solutions does the equation, $x^3 + x^2 + x = x^4 - x$ have?

a. 0 b. 1 c. 2 d. 3 e. 4

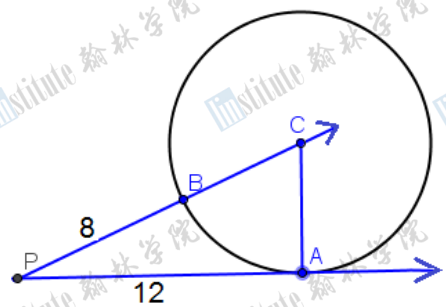
8. Let a, b and c each represent a distinct digit, and let the three digit number, abc, be divisible by both 9 and 11, and the two digit number, ac, be divisible by 8. Determine a.

a. 3 b. 4 c. 5 d. 6 e. 7

9. Determine the area of the triangle formed by connecting the points (1, 2), (4, 7) and (6, 5).

a. 6.5 b. 8 c. 10.5 d. 15 e. 16

10. Given a circle centered at C and a point P outside the circle. Let two rays emanate from P such that one of them is tangent to the circle at point A and the other passes through C, the circle's center. Let B be the intersection of the circle and $\overline{PC}$.
If $PA = 12$ and $PB = 8$, what is the diameter of the circle?



a. 5 b. $2\sqrt{20}$ c. 10 d. 14 e. none of these.

11. If $a = \log_2 5$ and $b = \log_5 2$, then which of these expressions simplifies to 1.

a. $a - b$ b. ab c. $\dfrac{ab}{a+b}$ d. $\dfrac{a-1}{b}$ e. $\dfrac{b+1}{a}$

12. Let $f(x) = \dfrac{x}{x+1}$ and $g(x) = \sqrt{x+1}$. Simplify $f\left(g^{-1}\left(\dfrac{1}{t-1}\right)\right)$.

a. $-t^2$ b. $\dfrac{t^2-1}{t^2}$ c. $\sqrt{\dfrac{t-1}{t}}$ d. $\dfrac{\sqrt{t}}{\sqrt{t}+1}$ e. $2t - t^2$

13. Let ϕ be the positive solution to $\phi - \frac{1}{\phi} = 1$. Which of the following is equivalent to ϕ^7?

a. $7\phi + 6$ b. $8\phi - 5$ c. $8\phi + 5$ d. $13\phi + 8$ e. $21\phi + 13$

14. Determine the larger value for $x + y$ where $x^2 + y^2 = 100$ and $x - y = 6$.

a. 10 b. $\sqrt{82}$ c. $3 + \sqrt{41}$ d. $\sqrt{164}$ e. $6 + \sqrt{164}$

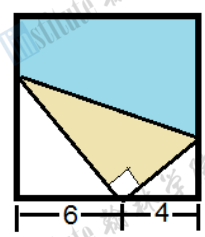
15. If 7^{2016} is divided by 5, what is its remainder?

a. 0 b. 1 c. 2 d. 3 e. 4

16. Peter has to drive 120 miles to get to his destination. He drives 90 mph for the first 30 miles. He then gets a ticket which took 15 minutes to process, and he continues his trip driving at 60 mph until he reaches his destination. What was his average speed for the entire trip?

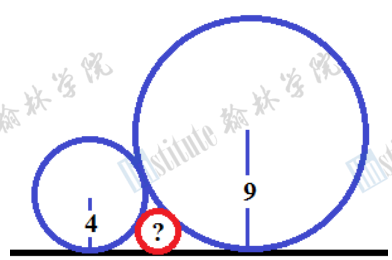
a. 57.6 mph b. 60 mph c. 61.25 mph d. 67.5 mph e. none of these

17. Figure on right has a 10 × 10 square which contains a right triangle whose three vertices touch the square's sides. If the triangle's hypotenuse has length $\sqrt{125}$, and the base of square is partitioned by a vertex into segments of measures 6 and 4, find the area of shaded region above the triangle.



a. $10\sqrt{3}$ b. 42 c. 45 d. 55 e. $15\sqrt{6}$

18. Three mutually tangent circles are each tangent to a line. If the two larger circles have radii of lengths 4 and 9 what is the length of the third circle's radius?



a. $\frac{4}{9}$ b. $\frac{25}{36}$ c. 1 d. 1.2 e. 1.44

19. If an equilateral triangle has the same area as a regular hexagon, what is the ratio of the hexagon's perimeter to the triangle's perimeter?

a. $\frac{\sqrt{6}}{6}$ b. $\frac{\sqrt{6}}{2}$ c. $\frac{3\sqrt{3}}{2}$ d. $\frac{2\sqrt{2}}{3}$ e. $\frac{\sqrt{6}}{3}$

20. Let $f(x) = \frac{x^2+9}{x+3}$ for what positive value of x is $f(x + 1) = 2x + 2$?

a. 1 b. $\frac{\sqrt{97}-9}{2}$ c. $3\sqrt{2}-4$ d. $\frac{\sqrt{39}-5}{2}$ e. $\sqrt{23}-4$

21. In Charlotte of the 200,000 voters 55% voted for candidate D. Of Asheville's 40,000 voters only 35% voted for candidate D, and in Boone only 30% of 10,000 voters supported "D". Of all voters, what percentage voted for "D"?

a. 50.8% b. 47⅓% c. 44⅔% d. 40% e. none of these

22. Determine the distance between the two intersections of line, $y = 2x - 6$ and parabola, $y = x^2 - 9$.

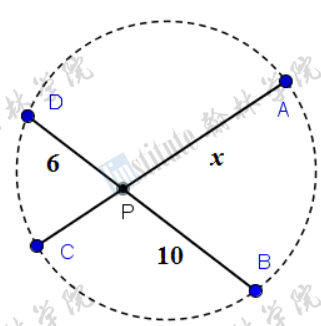
a. $\sqrt{42}$ b. $2\sqrt{17}$ c. $4\sqrt{5}$ d. 12 e. none of these

23. Determine the exact value of $\cos(\sin^{-1}(-0.6))$?

a. $\sqrt{0.4}$ b. $\frac{\pi}{6}$ c. 0.6 d. –0.8 e. none of these

24. Consider a circle with chords $\overline{CA}$ and $\overline{DB}$ intersecting at P. Let $CA = 17$, $DP = 6$, and $PB = 10$. If $CP < AP$, determine the measure of $\overline{AP}$.

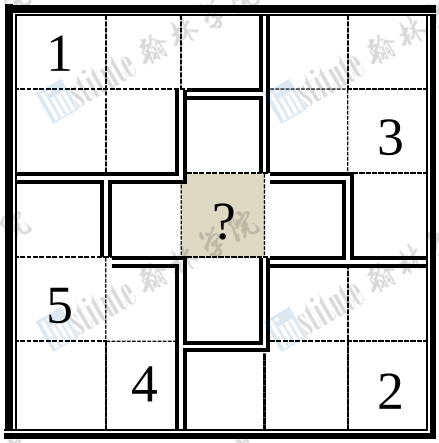
a. 5 b. $\frac{17+9\sqrt{21}}{2}$ c. $\frac{17+7\sqrt{3}}{2}$ d. 12 e. none of these



25. If you list all the natural numbers from 1 to 2016, how many do NOT contain the digit "7"?

a. 543 b. 1,315 c. 1,473 d. 1,743 e. none of these

26. In this Sudoku the digits 1-5 are placed in cells such that each digit appears once in every row, every column, and every group of cells.
Which digit is in the highlighted central cell?



a. 1 b. 2 c. 3 d. 4 e. 5

27. Let $(2+\sqrt{3})^5 = A + B\sqrt{3}$ where A and B are integers. Determine B.

a. 81 b. 209 c. 227 d. 362 e. none of these

28. How many integer values of k exist if $x^2 + kx + 12 > 0$ for all real numbers x?

a. infinte b. 6 c. 7 d. 13 e. 15

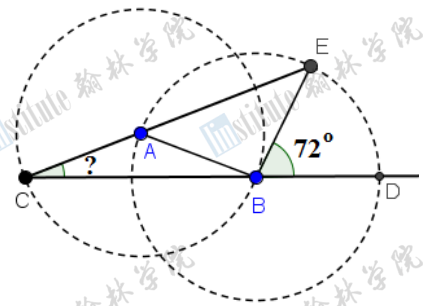
29. A tape 1mm thick and 5 cm wide is wrapped around a 5 cm wide circular core that has a 4 cm diameter. If you wrap the tape 50 times around this core, approximately how long is the tape?

a. 6 m b. 10 m c. 13 m d. 14 m e. 15 m

30. Solve $\log_2(x-7) + \log_2(x+3) = 3$.

a. $2+\sqrt{33}$ b. $2-\sqrt{33}$ c. 10 d. $4+\sqrt{432}$ e. $2+2\sqrt{7}$

31. Two circles centered at points A and B each contain the other's center. Point C is on one circle, and points E and D lie on the other circle. Points C, A and E are collinear as are points C, B and D.
If $\angle EBD = 72^\circ$ determine $\angle ACB$.



a. 18° b. 20° c. 24° d. 30° e. 36°

32. Under what condition is $(x+a+3)^2 = x^2 + a^2 + 9$ true? Solve for x in terms of a.

a. $x = -(a+3)$ b. $x = \dfrac{-3a}{a+3}$ c. $x = \dfrac{3a}{3-a}$ d. $x = \dfrac{-(3+a)}{3a}$ e. none of these

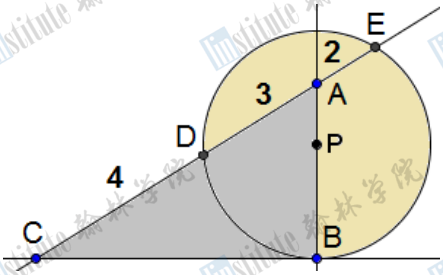
33. When the sun was 60º over the horizon Karen took note of the tip of the flagpole's shadow, later when the sun's angle was at 30º the shadow was 40 feet longer. How tall is the flagpole?

a. $20\sqrt{3}$ ft. b. 60 ft. c. $40\sqrt{3}$ ft. d. $\dfrac{40+20\sqrt{3}}{3}$ ft. e. none of these

34. A regular pentagon with sides of length one is in a rectangular box such that its vertices touch the sides of the box and one side is shared with the rectangle. Determine the area of one of the smaller triangles formed by the space between pentagon and rectangle.

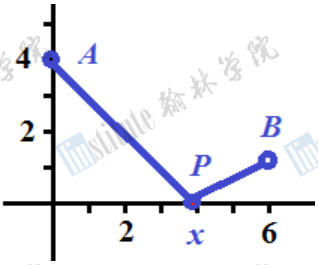
a. ½ tan(18º) b. ½ sin(18º) c. ½ cos(18º) d. ¼ sin(36º) e. ¼ tan(36º)

35. Consider a circle centered at P, and ΔABC where line $\overleftrightarrow{CB}$ is tangent to the circle. Let $\overleftrightarrow{AB}$ contain point P, and let $\overleftrightarrow{AC}$ intersect the circle at D and E.
If $CD = 4$, $DA = 3$ and $AE = 2$ determine the diameter of the circle.



a. 5 b. $\dfrac{19\sqrt{13}}{13}$ c. 5.4 d. $\dfrac{3\sqrt{13}}{2}$ e. $\dfrac{20\sqrt{13}}{13}$

36. The two points, A (0, 4) and B (6, 1), are on the xy-plane. Find the point, P (x, 0), on the x-axis such that the value of AP + PB is at a minimum.



a. $x = 3$ b. $x = 3.8$ c. $x = 4$ d. $x = 4.2$ e. $x = 4.8$

37. Express $\sqrt{46-6\sqrt{5}}+\sqrt{6+2\sqrt{5}}$ in "$A + B\sqrt{5}$" form.

a. $2+3\sqrt{5}$ b. $5+2\sqrt{5}$ c. $6\sqrt{5}$ d. $4\sqrt{5}$ e. 9

38. If $2^x = x^2 + 2$ has a solution into which of these intervals does it fall?

a. $x < 4$ b. $4 \le x < 4.5$ c. $4.5 \le x < 5$ d. $5 \le x$ e. No solution

Problems 39 & 40 address this cross number puzzle. Each cell contains one digit and k and n are natural numbers. Digits may appear more than once. Solve the puzzle then answer the questions. .

Across
1. Sum of all integers from 1 to n.
4. k^3
5. A factorial
Down
1. $(n-k)^2$
2. A power of 2
3. Number of 4 letter permutations using the word "ANSWER".

1	2	3
4		
5		

39. What is the value of $n+k$?

a. 24 or less b. 25 c. 27 d. 29 e. 30 or greater

40. Which digit listed below was NOT represented in the puzzle grid?

a. 1 b. 3 c. 4 d. 5 e. 6