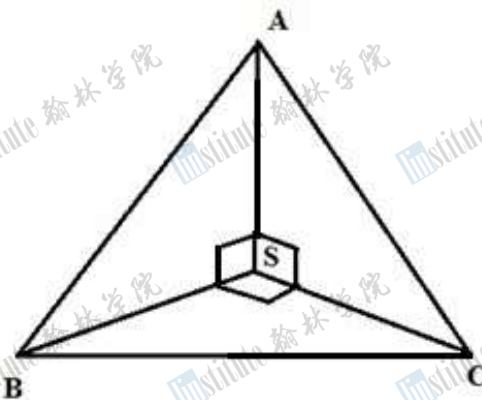


STATE MATH CONTEST-----Level III 2015

- Let $x = \log_3 4 \cdot \log_4 5 \cdot \log_5 6 \cdots \log_{2014} 2015$. Which of the following is true?
A. $x < 3$
B. $3 < x < 4$
C. $4 < x < 5$
D. $5 < x < 6$
E. $6 < x < 7$
- Let x, y, z be distinct real numbers with $x < 0$. If $x^2 - 2xy + z^2 = 0$ and $yz > x^2$, then the relation between x, y and z is given by
A. $x < y < z$
B. $y < z < x$
C. $z < x < y$
D. $x < z < y$
E. $y < x < z$
- Find the real solutions of the equation $3^{2x^2-7x+3} = 4^{x^2-x-6}$
A. 3 only
B. 3 and -98
C. 3 and 121
D. 3 and $\frac{1+2\log_3 4}{2-\log_3 4}$
E. 3 and $\frac{1+2\log_4 3}{1-\log_4 3}$
- Let f be defined by the relation: $f(3n) = n + f(3n - 3)$ where n is a positive integer greater than 1 and $f(3n) = 1$ when $n = 1$. Find the value of $f(12)$.
A. 12
B. 10
C. 20
D. 30
E. None of these
- Let a, b, c, d be real numbers such that $a + b + c + d = 9$ and $a^2 + b^2 + c^2 + d^2 = 27$. Find the maximum value of d .
A. 3.75
B. $3\sqrt{3}$
C. 4.50
D. $3\sqrt{2}$
E. 1.25
- Suppose x, y are positive integers that satisfy the equation $y^2 + 3x^2y^2 = 30x^2 + 517$. Find the value of $3x + 2y$.
A. 36
B. 26
C. 23
D. 22
E. 20
- A city park has a monument with a tetrahedron shape as shown in figure.



Assume that the edges \overline{BS} , \overline{CS} , and \overline{AS} of the tetrahedron are perpendicular to each other at the vertex S , and the areas of the faces (triangles) $\triangle ABS$, $\triangle BCS$, and $\triangle ACS$ are 10, 11, and 12 square feet respectively. Find the area of triangle $\triangle ABC$.

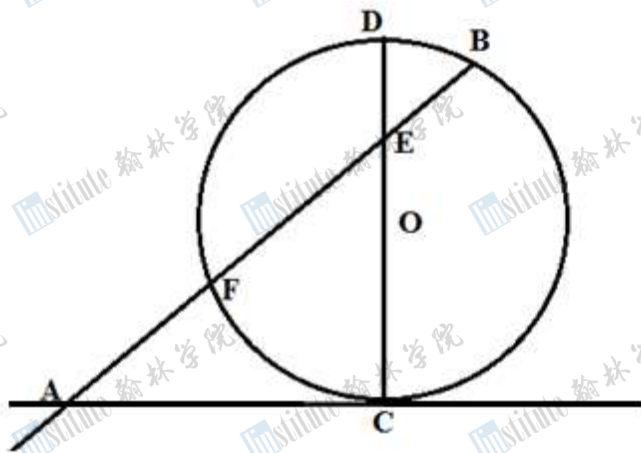
- A. 55
B. $\sqrt{365}$
C. 60
D. 66
E. $10\sqrt{3}$

8. Let a, b, c be three consecutive odd integers such that $a < b < c$. Consider the matrix

$$A = \begin{bmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{bmatrix}. \text{ Let } \det(A) \text{ denote the determinant of } A.$$

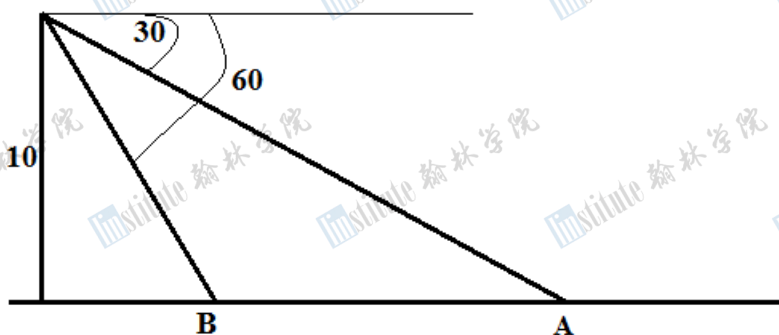
Find the value of $\det(A) + \det(A^2) + \cdots + \det(A^{2015})$.

- A. abc
 B. $(abc)^{2015}$
 C. $\frac{16}{15}(16^{2015} + 1)$
 D. $\frac{1}{15}(16^{2016} - 16)$
 E. $[b(c^2 - a)]^{2015}$
9. Let r, s, t, u be four real solutions of the equation $\left(x^2 - 2015x - \frac{2013}{2014}\right)^4 - 81 = 0$. Find the value of $2r + 2s + 2t + 2u$.
 A. 6480
 B. 3040
 C. 4030
 D. 6080
 E. 8060
10. If the three medians of a triangle ABC are 3, 4, and 5 feet, what is the area of the triangle ABC in square feet?
 A. 8
 B. 6
 C. 10
 D. 12
 E. None of these
11. The line \overleftrightarrow{AC} is tangent to a circle with center O at C . B is a point on the circle, and \overline{CD} passes through center O . \overline{AB} intersects the circle and \overline{CD} at F and E respectively. If $AF = 1$, $FE = 3$, and $EB = 2$. Find the radius of the circle.

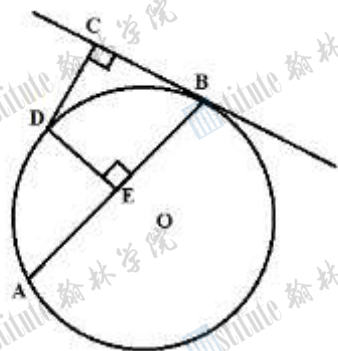


- A. $\frac{4\sqrt{10}}{5}$
 B. $\frac{3\sqrt{10}}{4}$
 C. $\frac{3\sqrt{5}}{5}$
 D. $\frac{4\sqrt{6}}{5}$
 E. $\frac{3\sqrt{6}}{4}$

12. The pilot of an airplane flying at an altitude of 10km sees two towns, A and B , directly in view ahead. Suppose the pilot's line of view makes angles of 30° and 60° respectively with the horizontal. How far apart are the towns A and B ?

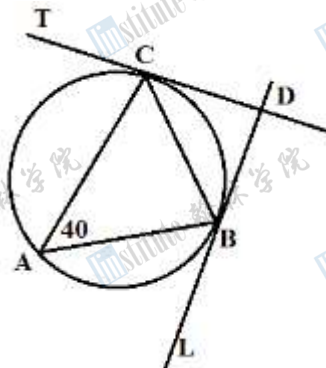


- A. $\frac{20\sqrt{3}}{3}$ B. 20 D. $10\sqrt{3}$
C. 10 E. None of these
13. Suppose \overline{CB} is tangent to the circle with center O at B . Let D be the midpoint of the arc AB . \overline{DE} is perpendicular to \overline{AB} at E , and \overline{DC} is perpendicular to \overline{BC} at C . Which of the following statements is NOT true?



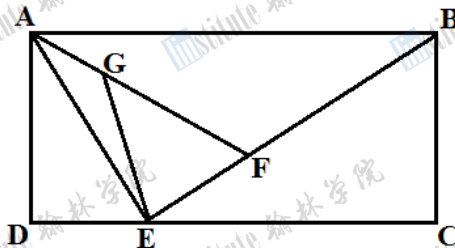
- (i) $DE = DC$
(ii) $BE = BC$
(iii) points D, C, E, B lie on a single circle
(iv) $AD = DB$
(v) $\angle EBC = 90^\circ$

- A. (iii) only C. (v) only E. (iv) only
B. (iii) and (v) only D. (iii) and (iv) only
14. In the figure, let \overline{TD} and \overline{LD} be tangent to the circle at C and B respectively. Suppose $\angle CAB = 40^\circ$. Find the measure of $\angle BDC$.



- A. 40° C. 100° E. None of these
B. 80° D. 120°

15. In rectangle $ABCD$, E is a point on side \overline{DC} , $AB = a$, and $BC = b$. If $FB = 2EF$ and $FG = 2GA$, then the area of triangle EFG is



- A. $\frac{1}{6}ab$ C. $\frac{1}{9}ab$ E. $\frac{1}{18}ab$
 B. $\frac{1}{8}ab$ D. $\frac{1}{12}ab$

16. Find the domain of the function

$$f(x) = \log_2 \left(\sqrt{\frac{(1-x)(x+3)}{(x+1)(x-3)}} \right)$$

- A. $(-1,1) \cup (1,3)$ D. $(-3,-1) \cup (1,3)$
 B. $(-\infty,-3) \cup (-1,1) \cup (3,\infty)$ E. $(-3,-1] \cup [1,3)$
 C. $[-3,-1) \cup (1,3]$

17. Find the area of the quadrilateral formed by the points of intersection of the curves $4x^2 + y^2 = 100$ and $9x^2 - y^2 = 108$.

- A. 96 C. 48 E. 80
 B. 84 D. 24

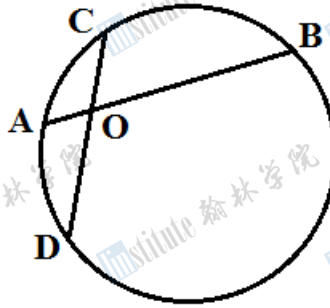
18. Let z_1 and z_2 be complex numbers such that $z_1 = 25 + i[(x-8)^2 + y^2]$ and $z_2 = x^2 + y^2 + 41i$, where x and y are real numbers. Find the value of $x^2 - y^2$ such that $z_1 = z_2$.

- A. -17 C. 6 E. None of these
 B. 9 D. -13

19. Find the sum of the real solutions of the equation $x^2 + 18x + 30 = 2\sqrt{x^2 + 18x + 45}$.

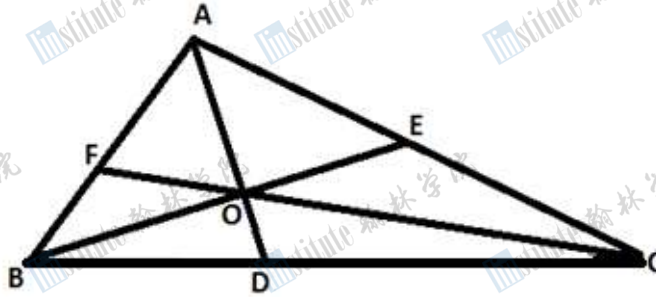
- A. -18 C. -20 E. -15
 B. 25 D. 16

20. Consider the circle shown in the figure. The chords \overline{AB} and \overline{DC} intersect at O such that $AO = 4\text{ cm}$ and $DC = 16\text{ cm}$. If $\overline{OC} = \overline{DO}$, find the length of \overline{AB} .



- A. 16 cm
 B. 20 cm
 C. $4\sqrt{5}\text{ cm}$
 D. $2\sqrt{38}\text{ cm}$
 E. 10 cm
21. Find the real solutions of the equation $2\ln(15 + e^2)^{(2x^2+2)} = \ln(e^2 + 15)^{(3x^2+8x-11)}$.
- A. $\{1, 4\}$
 B. $\{3, 5\}$
 C. $\{-5, -3\}$
 D. $\{2, 3\}$
 E. $\{-3, -2\}$

22. In $\triangle ABC$, \overline{BE} is a median and O is the midpoint of \overline{BE} . Draw \overline{AO} and extend it to meet \overline{BC} at D . Draw \overline{CO} and extend it to meet \overline{BA} at F . If $CO = 15$, $OF = 5$, and $AO = 12$, find the length of \overline{OD} .



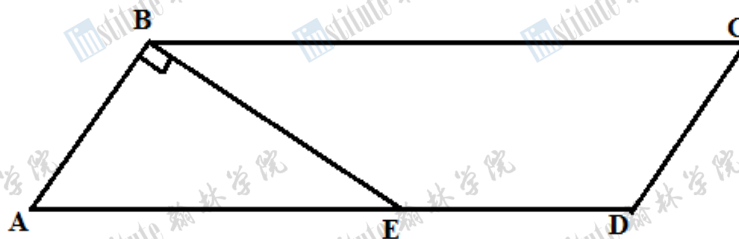
- A. 6.25
 B. 8
 C. 5
 D. 4
 E. None of these
23. If $|x| \leq M$ and $|x - y| \leq N$, where M and N are positive real numbers, then the greatest possible value of $|x^2 - y^2|$ is given by which of the following expressions?
- A. $M + N$
 B. MN
 C. $2MN + N^2$
 D. $MN + N^2$
 E. $M^2 + N^2$

24. Solve the following equation for real values of x .

$$\frac{\frac{x^{2015}}{2} - 61x^{1009} + \frac{121}{2}x^3}{(3x^{504} + 3x)\left(\frac{x^{505}}{6} + \frac{11}{6}x^2\right)} = 0$$

- A. $\{1, 11\}$
 B. $\{-1, \sqrt[503]{11}\}$
 C. $\{1, \sqrt[503]{11}\}$
 D. $\{-\sqrt[2015]{11}, \sqrt[503]{11}\}$
 E. $\{-1, 11\}$

25. In parallelogram $ABCD$, $\overline{AB} = \overline{BE} = \overline{ED}$ and $\angle ABE = 90^\circ$. If the area of the parallelogram $ABCD$ is 16cm^2 , how long is the side \overline{AD} ?



- A. $4(1 + \sqrt{2})\text{cm}$
 B. $4(2 + 2\sqrt{2})\text{cm}$
 C. $4\sqrt{2 + \sqrt{2}}\text{cm}$
 D. $10(1 + \sqrt{2})\text{cm}$
 E. $5(\sqrt[4]{2} + \sqrt{2})\text{cm}$
26. Let z be a complex number and \bar{z} be its complex conjugate. Find the number of complex solutions of the equation $\bar{z} = 1/z$

- A. 2
 B. 4
 C. Infinitely many
 D. 8
 E. None of these
27. During clinical trials of a new drug intended to reduce the risk of a given disease, a sample of 100 persons was drawn from a susceptible population. Information on the adverse side effects of the drug has been collected by the researchers. Every participant has encountered at least one side effect. The following designations are implemented concerning the side effects:

X: Nausea Y: Headache Z: Abdominal Pain

The statistical data on the adverse side effects are displayed as follows:

Adverse Effect	X only	Y only	Z only	X and Y only	Y and Z only	X and Z only	X and Y and Z
Number of persons	25	20	30	5	5	5	10

Compute the probability that a participant chosen at random has Nausea or Headache adverse side effects.

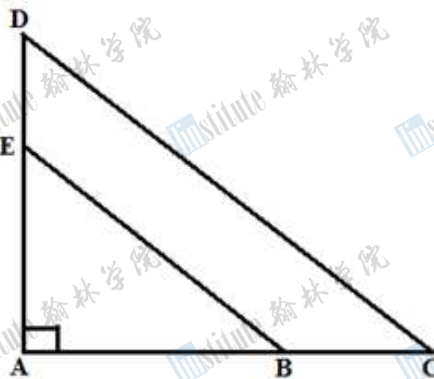
- A. 0.40
 B. 0.70
 C. 0.65
 D. 0.45
 E. None of these
28. The probabilities that a randomly selected freshman from a particular college needs remedial Mathematics is 30%, that (s)he needs remedial English is 40%, and that (s)he does NOT need either remedial course is 35%. Determine the probability that a freshman selected at random needs to take both a remedial Mathematics class and a remedial English class.

- A. 0.05
 B. 0.12
 C. 0.18
 D. 0.25
 E. 0.35

29. An insurance company estimates that 40% of policy holders who have only an auto policy will renew next year, 60% of policy holders who have only a homeowner policy will renew next year, and 80% of policy holders who have both an auto and a homeowner policy will renew at least one of those policies next year. Company records show that 65% of policy holders have an auto policy, 50% of policy holders have a homeowner policy, and 15% of policy holders have both an auto and a homeowner's policy. Calculate the percentage of policy holders that will renew at least one policy next year.

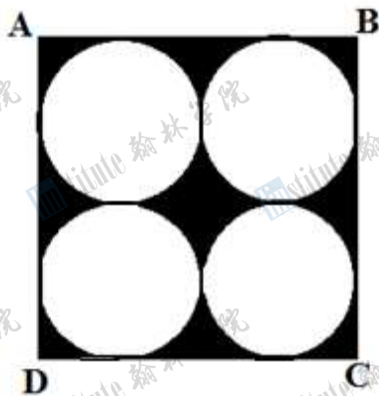
A. 20% C. 41% E. 60%
B. 29% D. 53%

30. In the figure below $\overline{BE} \parallel \overline{CD}$, $AE = x$, $\angle EAB = 90^\circ$, $BE = y$, $CD = 20$, $DE = 3$, $AB = 12$, $BC = 4$. Find the value of $x + y$.



A. 17 C. 37 E. None of these
B. 32 D. 28

31. In the figure below, the radius of each circle is 2 units. If a dart is thrown at random at the frame $ABCD$, find the probability that it will hit the shaded region.



A. $\frac{2-\pi}{2}$ C. $\frac{\pi}{4}$ E. $\frac{8-\pi}{8}$
B. 16π D. $\frac{4-\pi}{4}$

32. In the mythical country of Jamais, a charter flight charges each passenger a fare of \$500 plus \$10 for each unsold seat on the plane. Suppose the plane can hold a maximum of 200 passengers. Find the number of tickets that has to be sold in order to attain the maximum revenue for the flight.

- A. 75
B. 175
C. 200
D. 125
E. None of these

33. Suppose $C(x) = x^2 - 40x + 405$. Compute the minimum value of $C(x)$.

- A. 5
B. 15
C. 10
D. 0
E. 405

34. Find the area of a circle that passes through the points $P_1 = (1, -4)$ and $P_2 = (5, 2)$ such that the center of the circle is located on the line $L: x - 2y + 9 = 0$.

- A. 47π
B. 32π
C. 65π
D. 42π
E. None of these

35. Which of the following numbers solve the equation $4^{x^2} \cdot 2^{4x} = \frac{1}{64}$?

- A. $\{-i, i\}$
B. $\{-1 - i\sqrt{2}, -1 + i\sqrt{2}\}$
C. $\{-2i, 2i\}$
D. $\{1, -3\}$
E. $\{-i\sqrt{2}, i\sqrt{2}\}$

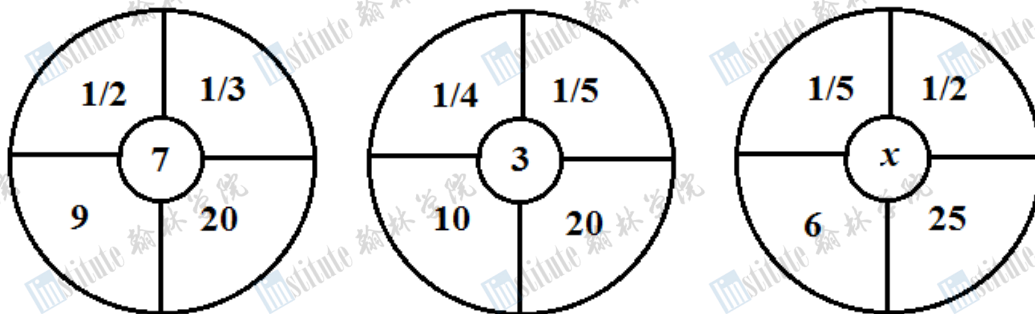
36. Find the value of k such that $f(x) = x^3 - kx^2 + kx + 2$ has the factor $(x - 2)$.

- A. 2
B. 10
C. 5
D. 15
E. 12

37. An urn contains 10 balls such that 4 of the balls are red and the rest are green. A second urn contains 16 red balls and n green balls. A single ball is drawn at random from each urn. The probability that the balls are of the same color is 0.44. Find the value of n – the number of green balls in the second urn.

- A. 4
B. 8
C. 12
D. 16
E. 2

38. Consider the following sequence of patterns:



Find the value of x .

- A. 3
B. 7
C. 5
D. 2
E. 1

39. The oscillations of an engineering device can be modeled by the function $y(t) = 1.5 \sin 5t - 0.5 \cos 5t$, where $y(t)$ is the distance from its equilibrium position, measured in feet and t is measured in seconds. The model can be written in the form $y(t) = A \sin(Bt + C)$. Find the values of A and C .

- A. $\left\{A = \frac{1}{3}\sqrt{10}, C = \tan^{-1}\left(\frac{1}{3}\right)\right\}$
- B. $\left\{A = \frac{1}{2}\sqrt{10}, C = \tan^{-1}\left(\frac{1}{3}\right)\right\}$
- C. $\left\{A = \frac{1}{3}\sqrt{10}, C = \tan^{-1}\left(\frac{1}{5}\right)\right\}$
- D. $\left\{A = \frac{1}{2}\sqrt{10}, C = -\tan^{-1}\left(\frac{1}{3}\right)\right\}$
- E. $\left\{A = \frac{1}{2}\sqrt{10}, C = -\tan^{-1}\left(\frac{1}{5}\right)\right\}$

40. Consider the triangle ABC with vertices $A = (0,0)$, $B = (4,0)$, and $C = (0,8)$. Let triangle $A'B'C'$ be the image of triangle ABC after the transformation $T(x,y) = \left(\frac{1}{2}x, \frac{1}{4}y\right)$. Find the area of triangle $A'B'C'$.

- A. 16
- B. 8
- C. 4
- D. 1
- E. 2