

**GEOMETRY**  
**State Mathematics Contest Finals**  
**April 28, 2011**

1. A ladder leans against a wall. The top of the ladder is  $8\text{ ft}$  above the ground. If the bottom of the ladder is then moved  $2\text{ ft}$  farther from the wall, the ladder will lie on the ground with its top touching the wall. How long is the ladder (in  $\text{ft}$ )?  
a. 13      b. 14      c. 15      d. 16      e. 17
2. A cube is inscribed in a sphere of radius  $1''$ . Find the surface area of the cube.  
a.  $4\text{ in}^2$       b.  $6\text{ in}^2$       c.  $8\text{ in}^2$       d.  $10\text{ in}^2$       e.  $12\text{ in}^2$
3. Points  $B$  and  $C$  lie on  $\overline{AD}$ . The length of  $\overline{AB}$  is 4 times the length of  $\overline{BD}$ , and the length of  $\overline{AC}$  is 9 times the length of  $\overline{CD}$ . The length of  $\overline{BC}$  is what fraction of the length of  $\overline{AD}$ ?  
a.  $\frac{1}{36}$       b.  $\frac{1}{13}$       c.  $\frac{1}{10}$       d.  $\frac{5}{36}$       e.  $\frac{1}{5}$
4. In the magic square, the sum of the three numbers in any row, column or diagonal is the same. The sum of the three numbers in any row is:

$2x$	3	2
		-3
0	$x$	

  
a. 0      b. 1      c. 3      d. 7      e. 9
5. A square and an isosceles right triangle each have a perimeter of 16 meters. To the nearest whole number, what is the absolute value of the difference between the number of square meters in their areas?  
a. 4      b. 5      c. 7      d. 8      e. 12
6. A rectangular floor measures  $a$  feet by  $b$  feet, where  $a$  and  $b$  are positive integers with  $b > a$ . An artist paints a rectangle on the floor with the sides of the rectangle parallel to the sides of the floor. The unpainted part of the floor forms a border of width 1 foot around the painted rectangle and occupies half the area of the entire floor. How many possibilities are there for the ordered pair  $(a, b)$ ?  
a. 1      b. 2      c. 3      d. 4      e. 5

7. The length of the hypotenuse of a right triangle is 4 and one of the acute angles has measure  $30^\circ$ . What is the radius of the circle with center at the vertex of the  $30^\circ$  angle which separates the triangle into two equal areas?

a.  $\sqrt{\frac{12\sqrt{3}}{\pi}}$       b.  $\sqrt{\frac{24\sqrt{3}}{\pi}}$       c.  $1+2\sqrt{3}$       d. 2.5      e.  $1+\frac{\pi}{2}$

8. The sides of a right triangle with integral lengths are in arithmetic progression. What is the length of the shortest side of such a triangle whose perimeter is greater than 2010?

a. 502      b. 504      c. 524      d. 612      e. 672

9. Point  $A = (2,3)$  is reflected over the  $x$ -axis to a point  $B$ . Then  $B$  is reflected across the line  $y = x$  to a point  $C$ . What is the area of triangle  $ABC$ ?

a. 12 units<sup>2</sup>      b. 14 units<sup>2</sup>      c. 15 units<sup>2</sup>      d. 17 units<sup>2</sup>      e. none of these

10. If the time is now 5 minutes past 12, when will the next time be such that the angle between the minute hand and the hour hand (in reverse order from now) is the same as it is now?

a.  $\frac{7}{12}$  of a minute before 1:00      d.  $10\frac{5}{11}$  of a minute past 1:00  
b.  $\frac{7}{12}$  of a minute past 12:55      e. none of these  
c.  $\frac{5}{11}$  of a minute past 1:00

11. A circle of radius 4 is inscribed in an equilateral triangle. The area inside the triangle but outside the circle can be written as  $p\sqrt{3} - q\pi$ . The value of  $p + q$  is

a. 22      b. 34      c. 40      d. 52      e. 64

12.  $P, A, B, C$ , and  $Q$  lie on a line in this order. If  $\frac{AC}{PQ} = \frac{3}{5}$ ,  $\frac{BC}{AC} = \frac{1}{3}$ , and  $PQ = 20$ , find  $AB$ .

a. 4      b. 5      c. 6      d. 8      e. 12

13. Given  $\triangle QPS$  with  $PQ = 4$ ,  $PS = 7$  and point  $R$  on  $\overline{QS}$  such that  $m\angle RPS = m\angle S$ . If the perimeter of  $\triangle PQR$  is 12, what is the perimeter of  $\triangle QPS$ ?
- a. 15      b. 19      c. 20      d. 21      e. 22
14. On a rectangular lot 50 ft by 120 ft, a man who walks 5 ft per second may cross diagonally from one corner to the opposite corner or walk along the perimeter of the lot. How much time (in seconds) does he save by walking diagonally?
- a. 8      b. 10      c. 16      d. 34      e. 42
15. The diagonal of a square is  $d$ . The perimeter of a second square which has four times the area of the first one is:
- a.  $4d$       b.  $\sqrt{2}d$       c.  $4\sqrt{2}d$       d.  $4d^2$       e. none of these
16.  $A$ ,  $B$  and  $C$  are the vertices of an isosceles right triangle with right angle at  $B$ . The area of this triangle is 1. Point  $E$  on side  $\overline{AC}$  of this triangle is such that segments  $\overline{AE}$  and  $\overline{AB}$  have the same length. Point  $D$  on side  $\overline{BC}$  of this triangle is such that the segment  $\overline{DE}$  is perpendicular to side  $\overline{AC}$ . The area of the triangle whose vertices are  $A$ ,  $D$  and  $C$  is:
- a.  $\frac{1}{2}$       b.  $\sqrt{2}$       c.  $\frac{1}{\sqrt{2}}$       d.  $2 - \sqrt{2}$       e.  $\sqrt{2} - 1$
17. In the diagram, a smaller square lies inside a larger square. The perimeter of the smaller square is 72 cm. The shaded area is  $160 \text{ cm}^2$ . The perimeter of the larger square, in cm, is



- a. 58      b. 88      c. 116      d. 121      e. 112
18. A rectangular piece of paper has dimensions 10" and 6". Two opposite sides of the paper are joined without overlap to form a cylinder of maximum height. Of the following, which is the best approximation of the radius of the base of the cylinder, in inches?
- a. 1      b. 2      c. 3      d. 5      e. 6

19. A circle is centered at the vertex of the right angle of an isosceles right triangle. The circle passes through both trisection points of the hypotenuse of the triangle. If the length of a radius of the circle is 10 units, then the area of the triangle, measured in square units, is
- a. 72      b. 84.5      c. 90      d. 98      e. 112.5
20. Jane never lies on Monday, Tuesday, Wednesday and Thursday. Jean always tells the truth on Monday, Friday, Saturday and Sunday. On the rest of the days, they may tell the truth or they may lie. Both say they lied yesterday. What day is today?
- a. Monday      b. Tuesday      c. Thursday      d. Friday      e. Sunday
21. Circle  $B$  passes through the center of circle  $A$  and is tangent to it. Circle  $C$  passes through the center of circle  $B$  and is tangent to it. What fraction of the area of circle  $A$  lies inside circle  $B$  but outside circle  $C$ ?
- a.  $\frac{3}{16}$       b.  $\frac{1}{4}$       c.  $\frac{3}{8}$       d.  $\frac{3}{4}$       e.  $\frac{5}{16}$
22. Suppose  $A$ ,  $B$ ,  $C$  and  $D$  are positive integers such that exactly one of the following inequalities is false. Which inequality is false?
- a.  $C < D$       b.  $A < B$       c.  $B + C > A + C$       d.  $A < B + C + D$       e.  $A + C < B + D$
23. Two support wires of length 25 ft and 30 ft respectively, are attached to the top of a pole. The 30 ft wire is attached to the ground 10 ft beyond where the 25 ft wire is attached (as measured from the base of the pole). How far from the base of the pole is the 25 ft wire attached to the ground?
- a. 1.25 ft      b. 6.25 ft      c. 7.75 ft      d. 8.75 ft      e. 18.75 ft
24. Find the area of the region, in square units, enclosed by the graph of  $|x| + |y| = 6$ .
- a. 18      b. 36      c. 72      d. 144      e. none of these
25. A line with slope equal to 1 and a line with slope equal to 2 intersect at point  $P(1,6)$ . The line with slope equal to 1 intersects the  $x$ -axis at  $Q$  and the line with slope equal to 2 intersects the  $x$ -axis at  $R$ . What is the area of  $\triangle PQR$ ?
- a. 6 units<sup>2</sup>      b. 9 units<sup>2</sup>      c. 12 units<sup>2</sup>      d. 15 units<sup>2</sup>      e. 18 units<sup>2</sup>



26. A wooden cube with edge length  $n$  units (where  $n$  is an integer greater than 2) is painted black all over. By slices parallel to its faces, the cube is cut into  $n^3$  smaller cubes each of unit edge length. If the number of smaller cubes with just one face painted black is equal to the number of smaller cubes completely free of paint, what is  $n$ ?

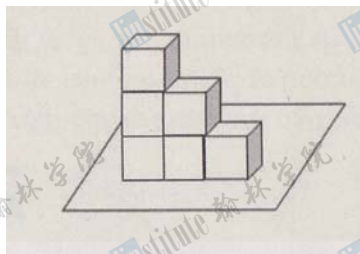
- a. 5      b. 6      c. 7      d. 8      e. none of these

27. A circle with center  $O$  and radius  $r$  is intersected at two distinct points by line  $\ell$ . Which of the following must be true?

- I. The distance between the points of intersection is less than  $r$ .  
 II. There is at least one point on  $\ell$  whose distance from  $O$  is less than  $r$ .  
 III. There is no point on  $\ell$  whose distance from  $O$  is less than  $\frac{1}{2}r$ .

- a. I only      b. II only      c. I and III only      d. II and III only      e. I, II and III

28. Six dice are stacked on the floor, as shown. On each die, the 1 is opposite the 6, the 2 is opposite the 5, and the 3 is opposite the 4. What is the maximum possible sum of numbers on the 21 visible faces?



- a. 78      b. 79      c. 81      d. 84      e. 89

29. Triangle  $ABC$ , with sides of length 5, 6, and 7, has one vertex on the positive  $x$ -axis, one on the positive  $y$ -axis, and one on the positive  $z$ -axis. Let  $O$  be the origin. What is the volume of the tetrahedron  $OABC$ ?

- a.  $\sqrt{85}$       b.  $\sqrt{90}$       c.  $\sqrt{95}$       d. 10      e.  $\sqrt{105}$

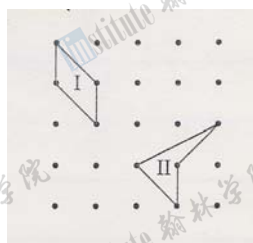
30. Seven different points are marked on the circumference of a circle. How many different triangles can be formed by connecting these points?

- a. 7      b. 14      c. 21      d. 22      e. 35

31. Three parallel lines  $\ell_1$ ,  $\ell_2$ , and  $\ell_3$  are drawn through the vertices  $A$ ,  $B$ , and  $C$  of a square  $ABCD$ . If the distance between  $\ell_1$  and  $\ell_2$  is 7 and between  $\ell_2$  and  $\ell_3$  is 12, then the area of  $ABCD$  equals

a. 95      b. 98      c. 156      d. 193      e. 288

32. Consider these two geoboard quadrilaterals. Which of the following statements is true?



- The area of quadrilateral I is more than the area of quadrilateral II.
- The area of quadrilateral I is less than the area of quadrilateral II.
- The quadrilaterals have the same area and the same perimeter.
- The quadrilaterals have the same area, but the perimeter of I is more than the perimeter of II.
- The quadrilaterals have the same area, but the perimeter of I is less than the perimeter of II.

33. An aquarium has a rectangular base that measures 100 cm by 40 cm and has a height of 50 cm. The aquarium is filled with water to a depth of 37 cm. A rock with volume  $1000 \text{ cm}^3$  is then placed in the aquarium and completely submerged. By how many centimeters does the water level rise?

a. 0.25      b. 0.5      c. 1      d. 1.25      e. 2.5

34. Point  $P$  is 6 units from the center of a circle of radius 10. Compute the number of chords with integral length that pass through  $P$ .

a. 2      b. 5      c. 7      d. 8      e. 10

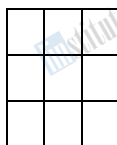
35. The measures (in degrees) of the interior angles of a convex hexagon form an arithmetic sequence of positive integers. Let  $m^\circ$  be the measure of the largest interior angle of the hexagon. The largest possible value of  $m^\circ$  is

a.  $165^\circ$       b.  $167^\circ$       c.  $170^\circ$       d.  $175^\circ$       e.  $179^\circ$

36. Suppose that the area of trapezoid  $ABCD$  is 72 square  $cm$ . If  $AB = 8cm$ ,  $CD = 16cm$  and  $m\angle ADC = 45^\circ$ , find  $BC$ .

- a.  $3\sqrt{10}$   $cm$       b.  $8$   $cm$       c.  $6\sqrt{2}$   $cm$       d.  $2\sqrt{5}$   $cm$       e.  $2\sqrt{10}$   $cm$

37. Three darts are thrown at the square dartboard shown, and each lands in a different small square. What is the probability that the 3 squares in which the 3 darts land form a horizontal, vertical or diagonal row?



- a.  $\frac{1}{7}$       b.  $\frac{1}{3}$       c.  $\frac{2}{9}$       d.  $\frac{2}{21}$       e.  $\frac{8}{27}$

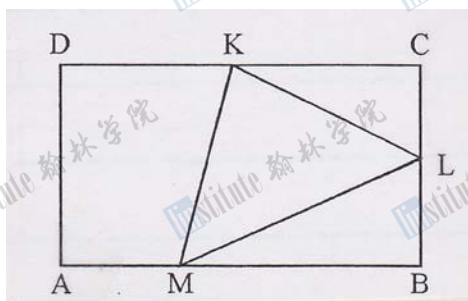
38. Point  $P$  is inside regular octagon  $ABCDEFGH$  so that triangle  $ABP$  is equilateral. How many degrees are in angle  $APC$ ?

- a.  $105^\circ$       b.  $112.5^\circ$       c.  $120^\circ$       d.  $125.5^\circ$       e.  $135^\circ$

39. A speaker talked for sixty minutes to a full auditorium. Twenty percent of the audience heard the entire talk and ten percent slept through the entire talk. Half of the remainder heard one third of the talk and the other half heard two thirds of the talk. What was the average number of minutes of the talk heard by members of the audience?

- a. 24      b. 27      c. 30      d. 33      e. 36

40. In the rectangle  $ABCD$ ,  $AB = 10$ ,  $BC = 6$ ,  $K$  is the midpoint of  $\overline{CD}$  and  $L$  is the midpoint of  $\overline{BC}$ . If the area of the triangle  $KLM$  is 12, what is  $AM$ ? (Note that the drawing may *not* be accurate.)



- a. 5      b. 6      c.  $3\sqrt{5}$       d. 7      e.  $\frac{1}{2}(\sqrt{5} + \sqrt{10})$

**ANSWER KEY**  
**2011 Geometry State Finals**  
**April 28, 2011**

1. E	11. E	21. A	31. D
2. C	12. D	22. A	32. E
3. C	13. B	23. D	33. A
4. C	14. A	24. C	34. D
5. B	15. C	25. B	35. D
6. B	16. E	26. D	36. E
7. A	17. B	27. B	37. D
8. B	18. A	28. E	38. B
9. C	19. C	29. C	39. D
10. C	20. D	30. E	40. D

Tie Breakers:

Best performance of three: 10, 20, 40

Sudden Death: 7, 8, 22, 27, 31