GEOMETRY
## STATE MATHEMATICS CONTEST FINALS
APRIL 29, 2010

1. Sara can go to the movies tonight provided she finishes all her homework and finds a way to get there. To get there she can either borrow her dad's car or she can get a ride with her friend Amy. If Sara isn't able to go to the movies, what conclusion can we draw?

    a. Amy couldn't give her a ride or she couldn't borrow her dad's car.
    b. Sara didn't finish her homework and Amy couldn't give her a ride
    c. Sara didn't finish her homework and she couldn't find a way to get there.
    d. Sara didn't finish her homework or she could neither borrow her dad's car nor get a ride with Amy.
    e. Sara couldn't get a ride from Amy and she couldn't borrow her dad's car.

2. A perfectly spherical orange with radius r units is sliced into eighths by three perpendicular planes through its center so that the peel of each piece looks like the figure. Determine the distance around the edge of the peel in the picture (on the right) in terms of its radius r.
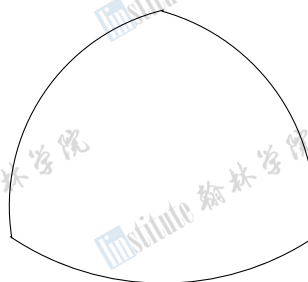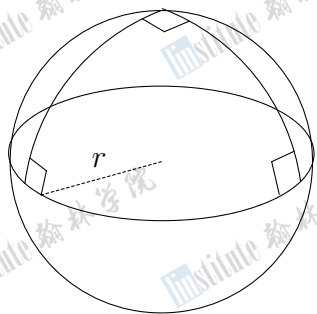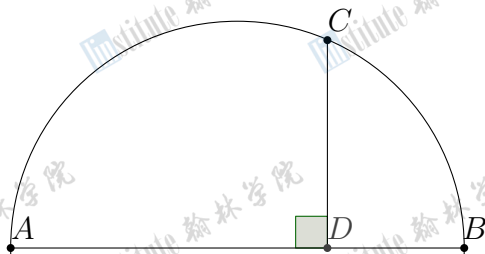

    a. $3r$ b. $\dfrac{3\pi r}{2}$ c. $3\pi r$ d. $\dfrac{2\pi r}{3}$ e. $2\pi r$

3. It takes 20 large ants to lift a cube of a certain material that measures 10 inches to a side. If all ants can lift the same amount of weight, how many more ants will be required to lift a cube of the same material that measures 11 inches to a side?

    a.1   b. 3   c. 6   d. 7   e. 9

4. In the diagram, $\overline{AB}$ is a diameter of a circle and $\overline{AB} \perp \overline{CD}$. If $AD = 1$ and $DB = a$, find CD.


a. $CD = 2a$ b. $CD = \sqrt{2}a$ c. $CD = \sqrt{1+a}$ d. $CD = \sqrt{1+a^2}$ e. $CD = \sqrt{a}$

5. The triangular numbers, $1, 3, 6, 10, \ldots$, are so-named because they occur as the number of points in an equilateral triangular arrangment. Which of the following is NOT a triangular number?

a. 210   b. 28   c. 465   d. 84   e. 55

6. A rectangular field is partitioned into four smaller rectangular fields. The areas of three of the fields are given. Find the missing area.
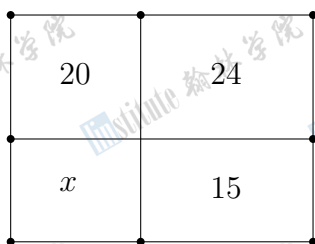
a. 11   b. 12   c. $\dfrac{25}{2}$ d. $\dfrac{21}{2}$ e. $\dfrac{19}{2}$

7. The diagram shows a regular heptagon, a regular hexagon and a regular pentagon all with equal side length. Find $m\angle 1$.
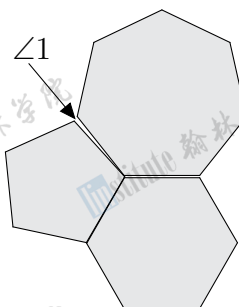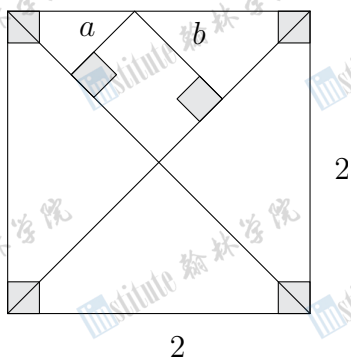
a. $\frac{24}{7}^{\circ}$ b. $\frac{74}{21}^{\circ}$ c. $\frac{70}{21}^{\circ}$ d. $\frac{23}{7}^{\circ}$ e. $\frac{22}{7}^{\circ}$

8. In the figure, the segments of length a and b lie on perpendiculars to the diagonals of a square of side length 2. Find $a + b$.

a. $\sqrt{2} - 1$ b. $\sqrt{2}$ c. $\frac{\sqrt{2}}{2}$ d. $\sqrt{\sqrt{2}}$ e. $\sqrt{\sqrt{2} + 1}$

9. A triangle has integer side lengths. If two of the lengths are 13 and 17 find the minimum possible value of the perimeter, m, and the maximum possible value of the perimeter M.

a. $m = 34$, $M = 60$.
b. $m = 35$, $M = 60$.
c. $m = 34$, $M = 61$.
d. $m = 35$, $M = 59$.
e. $m = 34$, $M = 59$.

10. Find the statement that is equivalent to the converse of "if P then Q."

    a. not P implies not Q
    b. if not Q then not P
    c. P implies not Q
    d. P if and only if Q
    e. not P if Q

11. The following four diagrams each depict two triangles. In which diagram can we NOT conclude that the triangles are congruent? Note that the pictures may not be to scale angles and lengths that are not explicitly stated to be congruent must not be assumed to be. You may assume that angles and segments labeled with the same letters are congruent.
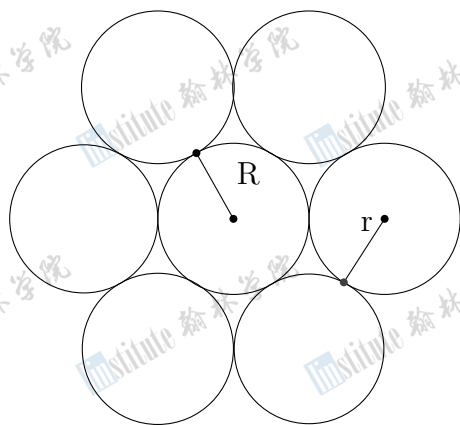

a.

b.
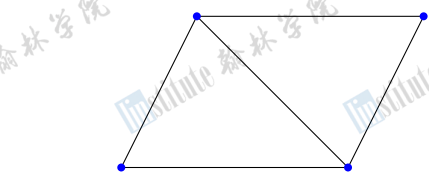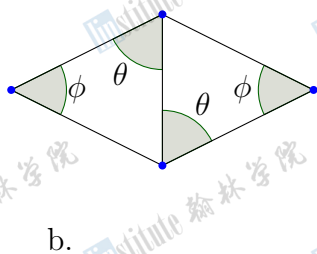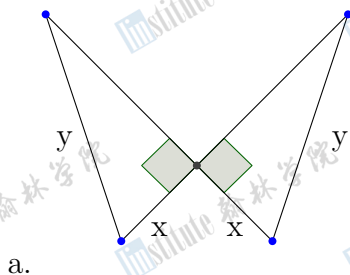
c. This quadrilateral is a parallelogram

d.
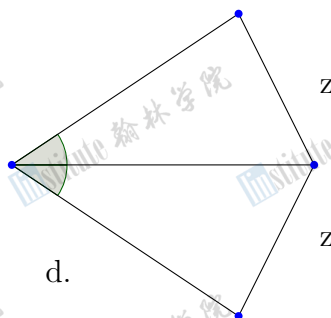
e. None of these.

12. A circle of radius R is surrounded by six circles of radius r arranged in a hexagonal pattern in such a way that each circle of radius r is tangent to two other circles of radius r and the center circle with radius R. Find the ratio $\dfrac{R}{r}$. Note that the picture may not be to scale.

   a. $\dfrac{\sqrt{2}}{2}$ b. $\sqrt{2}$ c. 1   d. $\dfrac{1+\sqrt{5}}{2}$ e. $\dfrac{\sqrt{3}}{2}$

13. A certain regular hexagon has area A and perimeter P. What is the perimeter of a regular hexagon with area $2A$?

   a. $2P$ b. $\sqrt{3}P$ c. $\dfrac{\sqrt{6}}{2}P$ d. $\sqrt{6}P$ e. $\sqrt{2}P$
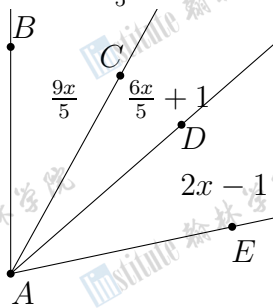
14. A right triangle has hypoteneuse with length 10 units and area 24 square units. Find its perimeter.

   a. 22   b. 24   c. 18   d. 16   e. 20

15. In triangle ΔABC the segment $\overline{AD}$ bisects the angle $\angle A$ and the point D lies on the segment $\overline{BC}$. Find BD if $AB = 6$, $AC = 4$ and $DC = 3$.
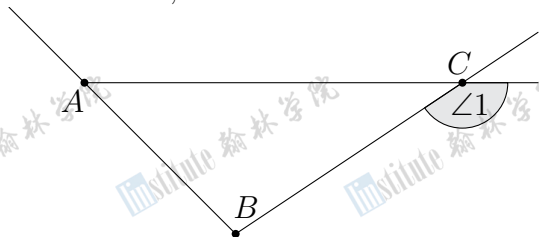
   a. $\dfrac{9}{2}$ b. 3   c. $\dfrac{5}{2}$ d. 8   e. 7

16. In the diagram, $\angle DAB \cong \angle EAC$ and m$\angle EAD = 2x - 1$, m$\angle CAD = \frac{6x}{5} + 1$ and m$\angle BAC = \frac{9x}{5}$. Find x.

a. 2   b. 3   c. 4   d. 5   e. 6

17. In $\triangle ABC$, $AB = BC$. If m$\angle 1 = 140°$, find m$\angle B$.

a. 40°   b. 90°   c. 100°   d. 140°   e. 80°
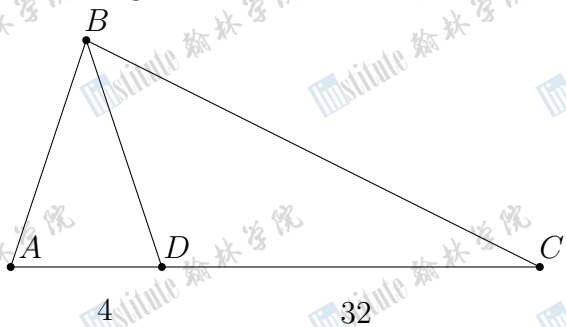
18. The exterior angles of a regular polygon are 40°. This polygon must be a

a. heptagon   b. nonagon   c. dodecagon   d. hexagon   e. octagon

19. If a square of sheet metal with side length s is rolled up to form a right circular cylinder, then the volume of this cylinder in cubic units is

a. $\dfrac{s^3}{4\pi}$ b. $\dfrac{s^3}{2\pi}$ c. $\dfrac{s^3\pi}{2}$ d. $\dfrac{s^3}{\pi}$ e. $\dfrac{2s^3}{\pi}$

20. Find AB given that $AC = BC$, $AB = BD$, $AD = 4$ and $CD = 32$.
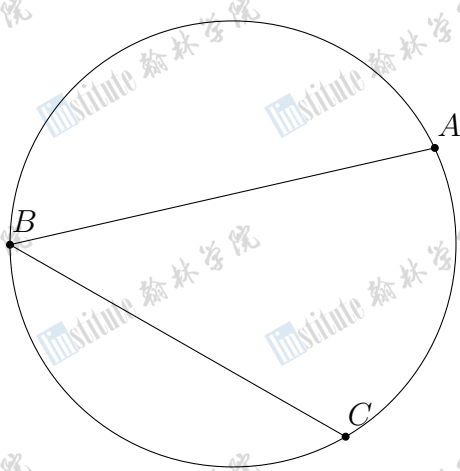

a.10   b. 9   c. 12   d. 8   e. 7

21. Find m$\angle ABC$ given that the measure of $\overset{\frown}{AB}$ is 140° and the measure of $\overset{\frown}{BC}$ is 130°.

In the figure $\overset{\frown}{ACB}$ is a major arc of the circle.
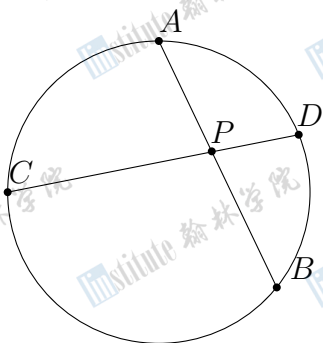

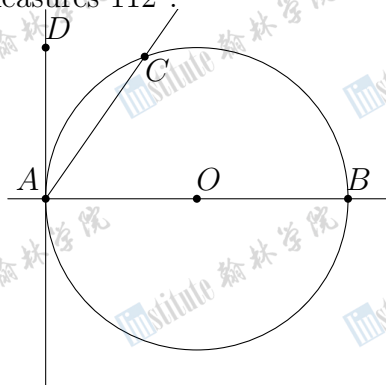a. 90°   b. 45°   c. 30°   d. 60°   e. 40°

22. If $\overline{AB}$ and $\overline{CD}$ are chords of a circle meeting at the point P interior to the circle and if $AP = 6$, $AB = 18$, and $PD = 4$, find CP.
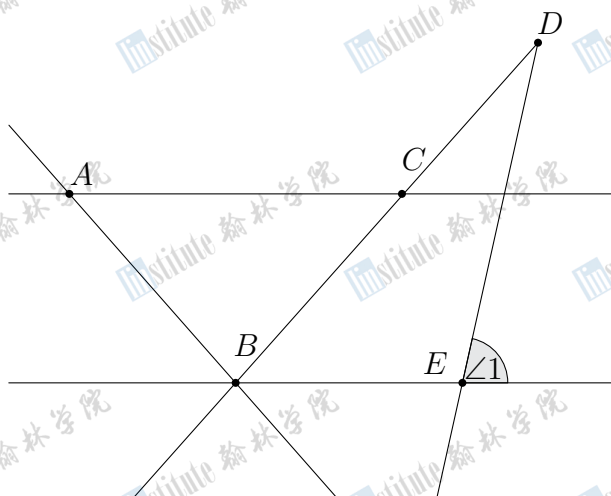

a. 4.5   b. 18   c. 16   d. 14   e. 6.5

23. Find m$\angle CAD$ in the diagram given that O is the center of the given circle and that $\overset{\frown}{BC}$ measures 112°.


a. 44°   b. 40°   c. 54°   d. 64°   e. 34°

24. Find m∠1 given that $\overleftrightarrow{AC}\parallel\overleftrightarrow{BE}$, $AB = BC$, m∠$D = 20°$ and m∠$BAC = 35°$.

 a. 85° b. 75° c. 65° d. 55° e. 45°

25. In the diagram, two semicircles with diameters x and $2x$, respectively are positioned along a diameter of a circle with diameter $3x$. If a point is chosen at random from the interior of the largest semicircle and its diameter, the probability that the point lies in the shaded region is
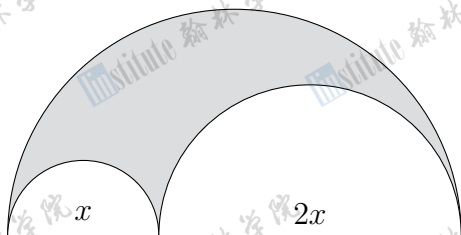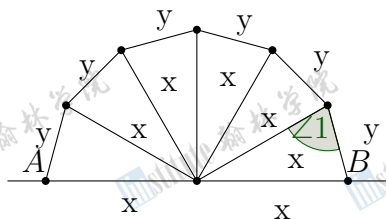

 a. $\dfrac{31}{36}$ b. $\dfrac{15}{36}$ c. $\dfrac{7}{9}$ d. $\dfrac{4}{9}$ e. $\dfrac{17}{36}$

26. Arrange the numbers x, y and z in increasing order, where $x = m\angle A$, $y = m\angle B$ and $z = m\angle C$.

13 / 12   13 / 13   13 / 13

5   5   4
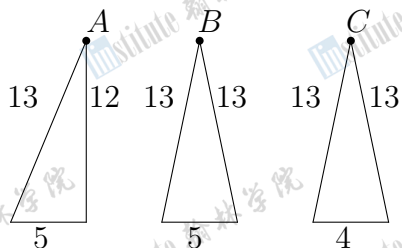
a. $x < y < z$ b. $x < z < y$ c. $z < x < y$ d. $y < z < x$ e. $z < y < x$

27. In the figure with line $\overleftrightarrow{AB}$ all segments labeled x are congruent and all segments labeled y are congruent. Find $m\angle 1$.
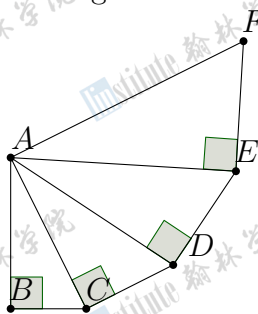

a. $80°$ b. $75°$ c. $70°$ d. $65°$ e. $60°$

28. Define the distance between two points, $P(x_1, y_1)$ and $Q(x_2, y_2)$, in the plane by $d(P, Q) = |x_2 - x_1| + |y_2 - y_1|$. How many points $P(x, y)$ in the plane have integer coordinates and are less than 6 units away from $O(0, 0)$?

a. 45   b. 80   c. 61   d. 36   e. 50

29. In the figure $AB = 2$, $BC = 1$ and $\angle FAE \cong \angle EAD \cong \angle DAC \cong \angle CAB$. Find AF.
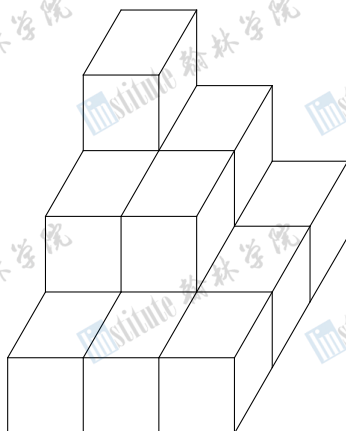
a. $\dfrac{5\sqrt{5}}{2\sqrt{2}}$ b. $\dfrac{5\sqrt{5}}{4}$ c. $\dfrac{25}{2\sqrt{2}}$ d. $\dfrac{25}{4\sqrt{2}}$ e. $\dfrac{25}{8}$

30. A child is stacking blocks and creates the pile shown in the diagram. If each face of a block has area 1, what is the total surface area of the figure?

a. 32 b. 84 c. 79 d. 42 e. 65

31. Which of the following expresses the perimeter of a right triangle with hypoteneuse measuring 16 units and with one angle measuring 32°?

a. $16(1 + \cos 32° + \sin 32°)$ units
b. $16(1 + \tan 32°)$ units
c. $16(\cos 32° + \sin 32°)$ units
d. $\dfrac{1}{16}(\cos 32° + \sin 32°)$ units
e. $\dfrac{1}{16}(1 + \tan 32°)$ units

32. Find the contrapositive of the statement: "If I attend the State Math Contest then I'm excused from school."

   a. "I attend the State Math Contest and I'm not excused from school."
   b. "If I don't attend the State Math Contest then I am not excused from school."
   c. "If I am not excused from school then I do not attend the State Math Contest."
   d. "I am not excused from school if I do not attend the State Math Contest."
   e. "I am excused from school if I attend the State Math Contest."

33. A circle is inscribed in a right triangle as indicated. Find the radius r of the inscribed circle if the area of the triangle is 6 and the length of the shorter leg of the triangle is 3.
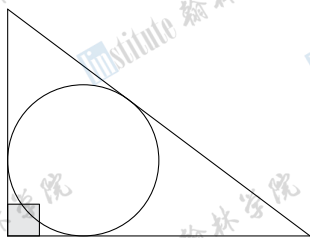
   a. 1   b. 2   c. $\sqrt{2}$ d. $\dfrac{\sqrt{3}}{2}$ e. $\dfrac{\sqrt{5}}{2}$.

34. A certain stop sign has the shape of a regular octagon with each side of length 1 foot. What is the distance from the bottom of the octagon to the top of the octagon?

   a. 2 ft   b. $1+\sqrt{2}$ ft   c. $1+\frac{1}{2}\sqrt{2}$ ft   d. $\frac{1}{2}(1+\sqrt{2})$ ft   e. $2-\sqrt{2}$ ft

35. If a circle of radius r is inscribed in an equilateral triangle which is itself inscribed in a circle of radius R, then $\frac{R}{r}$ is which of the following?

   a. $\dfrac{\sqrt{3}}{2}$ b. $\sqrt{2}$ c. $\sqrt{3}$ d. $\dfrac{\sqrt{2}}{2}$ e. None of these.

36. What can be said about the triangle with vertices $P(2,1)$, $Q(4,1)$ and $R(6,-3)$?

    a. It is an equilateral triangle.
    b. It is isoceles and the side opposite $\angle P$ is not congruent to either of the other sides.
    c. It is isoceles and the side opposite $\angle Q$ is not congruent to either of the other sides.
    d. It is isoceles and the side opposite $\angle R$ is not congruent to either of the other sides.
    e. It is a scalene triangle.

37. My old analog clock is fast. I know that the time it shows is within one hour of the correct time. At ten o'clock exactly, the minute hand and the hour hand are exactly opposite on the clock with the wrong time. How fast is this clock, to the nearest minute?

    a. 19 minutes   b. 20 minutes   c. 21 minutes   d. 22 minutes   e. 23 minutes

38. Determine the area of the polygon $ABCDE$ in square units.
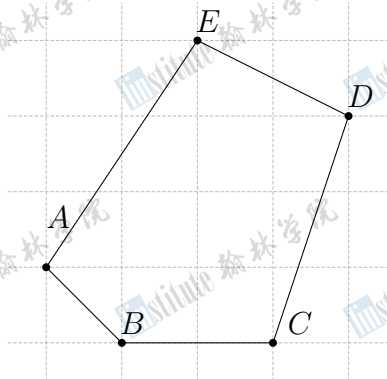
    a. 13   b. 11   c. 12   d. 10   e. 12.5

39. In the isoceles triangle $\triangle ABC$ $\overline{AB}$ is twice as long as $\overline{BC}$. If the perimeter of $\triangle ABC$ is 60, then AB is

    a. 24   b. 12   c. 15   d. 30   e. not uniquely determined

40. Which statement about quadrilaterals is false?

a. The sum of the measures of the interior angles of every quadrilateral is 360°.
b. Every square is a rhombus.
c. There is a trapezoid that is not a parallelogram.
d. There is a rhombus that is not a parallelogram.
e. The diagonals of every rhombus meet at right angles.