Algebra I
State Mathematics Contest Finals, April 29, 2010

1. The line $y = mx - 5$ passes through the point (2, 7). Find its slope m.

   a) –1   b) 6   c) 1   d) 12   e) 8.5

2. Let p and q be prime numbers such that $p < q$, $p + q = 32$, and $pq > 100$. Find p.

   a) 3   b) 5   c) 11   d) 13   e) 19

3. Adam is running a 10 kilometer race. Initially he is running at 10 km/hr then he gets tired and walks the remainder of the course at 6 km/hr. How long did it take him to complete the entire race if he spent half his time walking?

   a) 60 min.   b) 75 min.   c) 83.5 min.   d) 90 min.   e) none of these

4. For what value of c will $x(x - 4) = c$ have only one real solution?

   a) 0   b) 2   c) –4   d) 4   e) 16

5. If the points (1, 3), (5, 5), and (4r , r) are colinear, what is the value of r ?

   a) $-\dfrac{5}{2}$ b) $-\dfrac{1}{7}$ c) 0   d) $\dfrac{7}{5}$ e) $\dfrac{5}{3}$

6. $\dfrac{5 - \sqrt{3}}{1 + \sqrt{3}}$ can be rewritten as $a + b\sqrt{3}$ where a and b are rational numbers. Calculate $a+b$.

   a) –4   b) –2   c) –1   d) $\dfrac{1}{2}$ e) 2

7. Letters have been substituted for digits in the phrase "NOW + NO = HELP." Each letter

represents a unique digit; different letters represent different digits. If "O" is odd, "P" is prime, and "W" > "O", what is "WON + ONE"?

    a) 969   b) 959   c) 839   d) 829   e) 749

8.   If $x + 2y = a$ and $3x + 4y = b$ what does $x + y$ equal?

    a) $\dfrac{2a - b}{2}$ b) $\dfrac{b + a}{2}$ c) $\dfrac{3a - 2b}{4}$ d) $a + b$ e) $\dfrac{b - a}{2}$

9.   Four pirates come across a treasure and decide to divide their loot according to this plan: Ahab gets 20% of the treasure, Barnaby gets 30% of the loot that remains, Crusty gets 40% of the remaining part, and the Captain pockets the rest. How much of the treasure did the captain get?

    a) 10%   b) 25%   c) 33.6%   d) 50%   e) none of these

10.   If n is a whole number, which of the following expressions must also be a whole number?

    I. $\dfrac{n^2 + 5n}{2}$ II. $\dfrac{n^2 + 3n + 2}{3}$ III. $\dfrac{n^3 + 5n}{6}$

    a) I only   b) II only   c) I and II   d) I and III   e) II and III

11.   Divide the expression 7^{2010} by 5 and determine the remainder.

    a) 0   b) 1   c) 2   d) 3   e) 4

12. Given $f(x) = \dfrac{2x - 3}{x + 1}$, solve for a where $f(3a - 2) = 1$.

    a) 2   b) 4   c) $\dfrac{1}{2}$ d) 1   e) none of these

13.   Given $(3x - y)^2 = 3x^2 + 2xy - y^2$ where $x \neq 0$, find the solution set for $\dfrac{y}{x}$.

a) $\{1, 3\}$ b) $\{2 \pm \sqrt{2}\}$ c) $\{1 \pm \sqrt{7}\}$ d) $\{0, 3\}$ e) no solution

14. Rank the numbers $A = 2^{1/3}$, $B = 3^{1/4}$ and $C = 5^{1/6}$ from smallest to largest.

    a) $A < B < C$ b) $A < C < B$ c) $B < A < C$ d) $B < C < A$ e) $C < B < A$

15. How many real solutions does the equation $2 \cdot |2x + 1| - 1 = x^2$ have?

    a) 0   b) 1   c) 2   d) 3   e) 4

16. Let "*abc*" be a three digit number where *a, b,* and *c* are the digits. If the sum of its digits is 11, and if $ab - c = 11$ where "*ab*" is the two digit number, find the value of $a^2 + b^2 + c^2$.

    a) 51   b) 61   c) 65   d) 73   e) 85

17. 12 people got together for a meeting, and everyone shook hands with everyone else. How many handshakes were there?

    a) 55   b) 66   c) 72   d) 78   e) 132

18. At 3 o'clock the angle between the minute-hand and hour-hand of an analog clock measures 90°. How much time will elapse before the two hands form the next angle that measures 90°?

    a) 30 minutes   b) $32\frac{1}{2}$ minutes   c) $32\frac{8}{11}$ minutes   d) 65 minutes   e) none of these

19. A rectangular field that is twice as long as it is wide is modified such that the length is decreased by 10 meters and the width is increased by 10 meters. The area of the resulting rectangle is 42 square meters larger than the original one. How long was the original field?

    a) 14.2 m   b) 18.4 m   c) 24.2 m   d) 28.4 m   e) none of these

20. Betty has an account with a balance of $2048, and she decides to spend half of the money in this account every month. She knows her kind old dad will deposit $512 into her account shortly

before the end of every month, so some of the money will be replenished. Which expression best describes the amount of money in her account at the end of the nth month?

a) $1024 + \dfrac{1024}{2^n}$ b) $2048 - \dfrac{1024}{2^n}$ c) $512n + \dfrac{2048}{2^n}$ d) $\dfrac{1536}{2^n} + 512$ e) $2048 - 512n$

21. Given the expression $(2x + 3)^3$, what is the coefficient of the x^2 term?

a) 12 b) 18 c) 36 d) 48 e) 72

22. A gentleman chats with a lady while on an afternoon walk, both of them love mathematical puzzles, and they hold the following conversation:

Gentleman: It's my 36th birthday today,
Lady: What a coincidence, that number is the product of my three children's ages.
Gentleman: If that is the product can you tell me the sum of their ages?
Lady: That sum coincides with the number of windows of yonder building.
Gentleman (*counting windows.*) I cannot determine their ages with that information.
Lady: I guess you do need to know that I am presently a mother of a 1-year old.
Gentleman: Now I know how old they are.

How old are the lady's three children?

a) 1, 6, 6 b) 1, 3, 12 c) 1, 4, 9 d) 1, 2, 18 e) 1, 1, 36

23. What is the sum of the first 30 consecutive even natural numbers?

a) 720 b) 480 c) 903 d) 1028 e) 930

24. Write 7.1616161616… as a rational number.

a) $\dfrac{709}{99}$ b) $\dfrac{2387}{333}$ c) $\dfrac{716}{99}$ d) $\dfrac{725}{99}$ e) none of these

25. I am a real number. If you square me then the result is equal to twice the original me plus a number larger than 25. Which of the following statements is NOT true?

a) I could be a positive odd integer.

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b) I could be a negative irrational.
c) My value could be smaller than –25.
d) My value could be between –4 and 4.
e) I could equal to 42.

26. A particle starts moving at 1 meter per second in the positive direction. After five seconds it instantaneously reverses its direction and doubles its speed. It continues this pattern, reversing direction and doubling its speed every 5 seconds. With respect to its original position, where does the particle end up after 40 seconds?

   a) –425   b) –405   c) 0   d) 215   e) none of these

27. At what temperature (in Fahrenheit) is the Fahrenheit value twice the Celsius value?
   Hint: $^{o}F = (9/5)\cdot\ ^{o}C + 32.$

   a) 100º   b) 160º   c) 212º   d) 360º   e) none of these

28. How many digits does the number 2^{100} have?

   a) 100   b) 32   c) 31   d) 30   e) 29

29. A dam has three spillways of different sizes. The largest can lower the water level one foot in an hour, the second one takes 2 hours and the smallest one takes 3 hours. If all the spillways are opened, how many hours will it take to lower the water level by 33 feet?

   a) 5.5 hrs   b) 15 hrs.   c) 16.5 hrs   d) 17.8 hrs   e) 18 hrs

30. Given the function $f(x) = 4x + 1$, find the solution set for x where $f(x^2) = (f(x))^2$.

   a) $\left\{-\frac{2}{3}, 0\right\}$ b) $\left\{-\frac{2}{5}, 0\right\}$ c) $\left\{-\frac{1}{3}, 0\right\}$ d) $\{0\}$ e) $\left\{0, \frac{2}{3}\right\}$

31. The line $ax + by = 1$ intersects the circle, $x^2 + y^2 = 1$, at point $(-1, 0)$ find the x-coordinate of the other point.

a) $\dfrac{b^2-1}{2}$ b) $\dfrac{2b}{b^2+1}$ c) $\dfrac{2b}{\sqrt{b^2+1}}$ d) $\dfrac{b^2-1}{b^2+1}$ e) $\dfrac{\sqrt{b^2+2b}}{b^2+1}$

32. Suppose you have a bowl with 11 chips numbered 1 through 11. If you take out two chips, what is the chance that the difference of the two values is greater than 5?

 a) $\dfrac{1}{2}$ b) $\dfrac{3}{11}$ c) $\dfrac{5}{11}$ d) $\dfrac{21}{55}$ e) $\dfrac{32}{55}$

33. Assume you have 17 silver-colored coins in your pocket (nickels, dimes, and quarters), and that the value of the quarters is twice the value of the remaining coins. How many quarters do you have?

 a) 4 b) 5 c) 6 d) 7 e) 8

34. If $9^x = 20$ then determine the value of $\sqrt{5} \cdot 27^x$.

 a) $15\sqrt{15}$ b) $30\sqrt{5}$ c) $50\sqrt{3}$ d) $100\sqrt{5}$ e) 200

35. Find the x intercept of the line that passes through the points $(3, 42)$ and $(8, 27)$.

 a) –42 b) –17 c) 13.8 d) 17 e) 42

36. In this "Mini-Sudoku" puzzle you fill in the grid so that every row, every column, and every 2x2 box contains the digits 1 through 4. Note, this puzzle may have multiple solutions. Determine the number of possible different solutions to this Sudoku.

 a) 1 b) 2 c) 3 d) 4 e) more than 4

37. Find the distance between the points of intersection of $x^2 + y^2 = 9$ and $y = 2x - 3$.

a) 3   b) $\dfrac{12}{5}$ c) $3\sqrt{2}$ d) $2\sqrt{5}$ e) $\dfrac{12\sqrt{5}}{5}$

38. Let $\dfrac{x^2 + ax + 6}{x + 1} = x + b$ for all positive values of x. What is the value of $2a - b$?

   a) 6   b) 7   c) 8   d) 9   e) 11

39. A right triangle has the property that the lengths of its sides form a geometric progression. (i.e. the ratio of shorter leg to the longer leg is the same as the ratio of the longer leg to the hypotenuse.) What is the ratio of the hypotenuse to the shorter leg?

   a) 2   b) $\dfrac{5}{3}$ c) $\dfrac{\sqrt{5}}{2}$ d) $\dfrac{1 + \sqrt{5}}{2}$ e) $\sqrt{\dfrac{1 + \sqrt{5}}{2}}$

40. The operation "$\otimes$" is defined on the positive rational numbers as $a \otimes b = \dfrac{2ab}{a+b}$. Solve for x, where $(x \otimes 2) \otimes x = 1$.

   a) $\dfrac{6}{7}$ b) $\dfrac{2}{3}$ c) $\dfrac{\sqrt{2}}{2}$ d) $\dfrac{\sqrt{2}-1}{2}$ e) none of these