multille m # " multine m # " minitule # # " multille m # " mutule mark 's multinu m # 3 mutilite # # 'S PR 而此此此新祥等発 multile # # 3 PE minitute ## # '\$ PK **ALGEBRA I** Ro **State Mathematics Contest Finals** A s.c. April 30, 2009 1. The lines $x = \frac{1}{4}y + a$ and $y = \frac{1}{4}x + b$ intersect at the point (1,2). What is a + b? matterne # # # B Institute # # 'S PE a. 0 (1, 1) b. $\frac{3}{4}$ (1, 1) (1, 2) (1 2. Define the operation * by x * y = 4x - 3y + xy, for all real numbers x and y. For how many real numbers y does 3*y = 12? a. 0 b. 1 c. 3 d. 4 e. more than 4 Ro $\frac{1}{2}$ 2 3. What is the probability that, of three people selected at random, at least two were a. $\frac{2}{7}$ b. $\frac{3}{7}$ c. $\frac{19}{49}$ d. $\frac{27}{49}$ e. $\frac{30}{49}$ Ro 4. *ABCD* and *DCFE* are coplanar rectangles with AB = 4, AC = 5 and BC = CF. What is the length of \overline{AF} ? mutilut # # 3 PS Y. What is the length of \overline{AF} ? a. $2\sqrt{13}$ b. $\sqrt{34}$ c. $\sqrt{41}$ d. 6 e. 7 5. A line with slope 2 intersects a line with slope 6 at the point (40,30). What is the distance between the *x*-intercepts of these lines? N. a. 8 b. 10 c. 12 d. 14 e. 16 each row and once in each column. What number will occupy the lower right-hand square? N. To complete the grid below, each of the digits 1 through 4 must occur once in 1 1 2 a. b. 2 mutate # K & K 2 3 mutule ## # '& PL stitute \$ # 3 PS Y. 3 80 c. 4 4 d. e. Cannot be determined Lo the the B to the W- 1/3 Ph to the W. B. P. the the the the to the the By Pho Ro 如秋 後

7. A large rectangle is partitioned into four rectangles by two segments parallel to its sides. The areas of three of the resulting sides. The areas of three of the resulting rectangles are shown. What is the area of the fourth rectangle?

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Institute # # 3 PS 柳颜秋诸郎 We 新林·省代 斯斯林場外 118 频 法 化 8. *Parade* is a supplement to many Sunday newspapers. In the past year, *Parade* introduced Numbrix *TM* puzzles for its readers. The goal is to fill in the grid so that the numbers 1-81 are listed in sequence, such that consecutive numbers matilule ## # '& R follow a horizontal or vertical path (no diagonals). Find the number to be inserted matime # # '8 into the square marked 'X'.

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9. Find $f\left(f\left(\frac{1}{2}\right)\right)$ where f(x) is defined as $f(x) = \begin{cases} 2x & \text{if } 0 \le x < 1\\ \frac{1}{2}x & \text{if } 1 \le x < 2 \end{cases}$ a. $\frac{1}{4}$

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d. $1\frac{1}{2}$ c. 1 myinne # # 'S R

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multilite m # * Institute # ** * multinu m X 3 Institute \$7 % Institute \$5 \$5 'S Institute \$7 5 10. Determine all values of c for which $2x^2 + 8x + c = 0$ has real solutions. a. c < -8 b. $c \le -8$ c. $c \le 8$ d. $c \ge 8$ e. $-8 \le c \le 8$ Y. 11. |b-a| = |b| - |a| is true: 而此此此新祥後 而如此他称林塔梯 maxinte # # 3 PE a. for all real values of a and b.b. if a and b are positive real numbers c. for no real values of *a* and *b*. d. If b = 0. e. If a = 0 or a = b. 12. If a > 0 and b > 0 and the triangle in the first quadrant bounded by the coordinate axes and the graph ar + by = 6 has area 6, there a^{t} axes and the graph ax + by = 6 has area 6, then ab =13. Alice and Bob play a game involving a circle whose circumference is divided by 12 equally-spaced points. The points are numbered clockwise. From 1:: 12. The points are numbered clockwise. Y. on point 12. Alice moves clockwise and Bob, counterclockwise. In a turn of the game, Alice moves 5 points clockwise and Bob moves 9 points counterclockwise. The game ends when they stop on the same point. How many turns will this take? 而此此版教祥等除 maximue ## # '& PL a. 6 b. 8 c. 12 d. 14 e. 24 14. If $\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$, then if $\begin{vmatrix} 2 & x \\ 3 & 5 \end{vmatrix} = \begin{vmatrix} x & 2 \\ 1 & 1 \end{vmatrix}$, x =Y. matilute 新林塔 a. -3 b. 1 c. 0 d. 3 e. 4 15. If a + b = 1, and $a^2 + b^2 = 2$, then the product of *a* and *b* equals a. $-\frac{1}{2}$ b. 1 c. $\frac{1}{2}$ d. $1\frac{1}{2}$ e. 2 16. If the binary operation * is defined for integers by a * b = a + b - ab, which of the following is (are) true? following is (are) true? * is commutative. I. Multule # # 3 19 III. * is associative III.There exists some integer which is an identity for *a. I onlyb. III onlyc. I and II onlyd. I and III onlye. I e. I, II and III

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25. In a magic square the sum of the numbers in any row, column, or diagonal is the Ro same. For the magic square below, the sum of the numbers in any row is 3 2x-3 Institute ## # '& PS minute.94 3 % 而以前用新花等除 b. 1 c. 3 d. 7 26. If x and y are positive numbers and the average of 4, 20 and x is equal to the average of y and 16, then the ratio x : y is: c: 1:1 situte d. 2:5 stitute e. 5:2 stitute # # # # Y. musitute # # b. 2:3:11118 新林塔 a. 3:2 27. The whole numbers *m* and *n* satisfy m + n = 20 and $\frac{1}{m} + \frac{1}{n} = \frac{5}{24}$. The product *mn* is a. 72 b. 36 c. 48 d. 96 e. 24 面射机机都林塔然 mythill # # 3 PS Ro $\int -a(x-2)^2 + c$ the value of b is: 28. If $y = a(x-2)^2 + c$ and y = (2x-5)(x-b) represent the same quadratic function, matinue # # ** b. $\frac{3}{2}$ 11111 c. $\frac{4}{5}$ 11111 d. $-\frac{5}{2}$ 11111 e. $\frac{8}{5}$ Y. From past experience she knows that 29. A computer software retailer has 1200 copies of a new software package to sell. \$\$***** Y. Half of them will sell right away at the original price she sets. II. Two-thirds of the remainder will sell later when their price is reduced by 40%, and III. The remaining copies will sell in a clearance sale at 75% off the In order to make a reasonable profit, the total sales revenue must be \$72,000. To the nearest cent, what original price should she set? original price. N. withte the a. \$60.01 b. \$75.01 c. \$79.13 d. \$80.90 e. \$240.01 Inditute # # '& R 而此此他新祥等院 mating # # '& R mutute # # # B Institute # # 'S PR mutule ## # & PL Y.

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multille m # " Institute \$7 the 'S Institute m # " multille m # 3 matinte m # " matitute # # 3 30. Jonathan sold two mountain bikes. He made a 30% profit on the first one and a Ro matine ### 50% profit on the second one. If his total profit was 45%, then what was the ratio of his cost for the first bike to his cost for the second bike? a. 1:2 b. 1:3 c. 1:5 d. 2:3 e. 2:5 而就批批新林塔際 31. Consider the following pattern Ro 面的机机称样等 institute # # What is the difference between the number of dots in the n^{th} figure and the number of dots in the $(n-1)^{\text{th}}$ figure? mutute # ** Ro d. $n^2 - 1$ c. 2*n*−1 e. $(n-1)^2$ a. 2n b. *n* − 1 mutute # # '& R Ro 32. Of the following numbers, the one that can be written in the form 3N, where N is an integer, is: d. 444,444 a. 44 c. 44,444 e. 4,444,444 b. 4,444 mating # # 'S PS it's the Ro 33. A box contains 4 fair coins and 6 biased coins. Whenever a fair coin is flipped, it comes up heads with probability 0.5. Whenever a biased coin is flipped, it comes up heads with probability 0.8. A coin is randomly chosen from the box and then flipped. What is the probability that it will come up heads? · % % Mylinte # # 'S e. 0.76 N. c. 0.68 a. 0.6 b. 0.64 d. 0.72 34. What is the value of k, so that the graph of 4x + ky = 8 is a line parallel to the line passing through the points (6,-2) and (4,-1)? e. 8 3 % Astitute the the " free a. -8 b. $\frac{1}{2}$ c. $-\frac{1}{2}$ d. 2 35. If |x-2| = p, where x < 2, then x - p =No. c. 2-2p mytitte ## # # a. b. matitute ## # '& K e. |2p-2|2p multitute 新林塔保 d. 2p - 2b. 2 而如此他都林塔帮 Y. to the the the Lo the the B Lo the the the to the the B. to the the B Ph Ro

multille m # 3 Institute ## # Institute m # " Institute \$7 % multine # # 3 matitute # # 3 36. Gail has an incredible coin changing machine. When she puts in a quarter, it returns five nickels; when she puts in a nickel, it returns five pennies: and the she puts in a penny, it returns five quarter. R returns five nickels; when she puts in a nickel, it returns five pennies; and when she puts in a penny, it returns five quarters. Gail starts with just one penny. repeatedly? «"a. \$3.63 b. \$5.13 c. \$6.30 d. \$7.45 e. \$9.07 而如此他都林塔 37. An unusual die has the numbers 2, 2, 3, 3, 5 and 8 on its six faces. Two of these dice are rolled and the two numbers on the top faces are added. How many different totals are possible? 面对机能称并接触 8 b. 9 d. 11 a. c. 10 e. 12 Y. 38. The terms a_1, a_2, a_3 form an arithmetic sequence whose sum is 18. The terms $a_1 + 1, a_2, a_3 + 2$, in that order, form a geometric sequence. Find the sum of all possible values of a_1 . mstitute ## # % a. **b** 2 d. 11% c. 7 Ro e. 13 % 0 39. When 60 minutes elapse on a correct clock, 62 minutes register on clock F (fast) and only 56 minutes register on clock S (slow). If later in the day clock F reads 8:00 and clock S reads 7:00, what was the correct time when the two clocks were originally set? mythute ## # '& PL 40. Ramona just completed writing her first book and numbered the pages. Numbering the book required 2649 digits. How many pages down N. mittute # # '& R 面对机作新林谱院 e. 929 muitute # # c. 919 d. 923 a. 913 b. 917 N. mutule ## # '& R Institute ## # 13 PR astitute ## # '% PK Institute # # 'S R Aritute the tot is the Astitute the the 'S PR N. 面的机能称林塔张 mutule # # 'S R Withthe the the to the avitute # # * * asitute # # '\$ 1% Astitute # # 13 19 N. Lo the the B to the W- 1/3 Ph to the bit of the to the the the to the the By Ro · 按林 法