ALGEBRA I
STATE MATHEMATICS CONTEST FINALS
MAY 1, 2008

1. Simplify: $\dfrac{1}{2-\dfrac{3}{4-\frac{5}{x}}}$.

a. $\dfrac{2x-3}{5(x-1)}$ b. $\dfrac{3x+4}{2(x-5)}$ c. $\dfrac{x+3}{2(x-4)}$ d. $\dfrac{4x-5}{5(x-2)}$ e. None of these

2. Find the last two digits of $(17)^{107}$.

a.21 b. 59 c. 83 d. 07 e. 73

3. Find $f(f(f(\frac{1}{7})))$ where $f(x)$ is defined as follows:

$$f(x) = \begin{cases} 2x, & \text{if } 0 \le x < \frac{1}{2}; \\ 2-2x, & \text{if } \frac{1}{2} \le x \le 1. \end{cases}$$

a. $\dfrac{1}{343}$ b. $\dfrac{8}{7}$ c. $\dfrac{6}{7}$ d. $\dfrac{1}{7}$ e. $\dfrac{8}{343}$

4. The graph of the line $Ax + By = C$ is perpendicular to the graph $y = .4x - 3$, with A and B relatively prime integers and $A > 0$. If the lines intersect at the point $(10, 1)$ find C.

a.26 b. 52 c. 5 d. 17 e. cannot be determined

5. Let $h(x) = 3x - 2$. Find $h^{-1}(4)$.

a. $\frac{1}{10}$ b. 10 c. $\frac{1}{2}$ d. 2 e. $\frac{1}{4}$

6. A drawer contains 2 black socks, 3 brown socks and 4 blue socks. If two socks are selected at random, what is the probability that they are the same color?

a. $\frac{5}{9}$ b. $\frac{2}{3}$ c. $\frac{1}{4}$ d. $\frac{5}{18}$ e. $\frac{5}{12}$

7. Suppose a and b are two non-zero real numbers with $a < b$. Which of the following statements must be true regarding a and b?

I. $\frac{1}{a} > \frac{1}{b}$ II. $a^2 < b^2$ III. $a < |b|$

a. I only b. II only c. III only d. I and II e. I and III

8. A university professor notices that the number of students in her class is the smallest number so that 1 student remains if the class is divided into groups of 2, 2 students remain when the class is divided into groups of 3, 3 students remain when groups of 4 are formed, 4 students remain when groups of 5 are formed and 5 students remain when groups of 6 form. If x is the number of students in this class, what is true of x?

a. $0 \leq x \leq 30$ b. $31 \leq x \leq 60$ c. $61 \leq x \leq 90$ d. $91 \leq x \leq 120$ e. $x \geq 121$

Problems 9-12 concern the function

$$f(x) = ax^2 + 3x - 4,$$

where a is a non-zero real number.

9. Find the equation of the line containing $(1, f(1))$ and $(2, f(2))$.

a. $(3a+3)x - y = 2a + 4$
b. $3x - y = 2a - 4$
c. $3ax - 3y = a + 2$
d. $(3a+3)x - y = 2a + 2$
e. $(a+1)x - 3y = a - 4$

10. Find the set of values of a for which $f\left(\frac{2}{a}\right) = 0$.

a. $\frac{5}{2}$ b. -1 c. 2 d. $\frac{4}{3}$ e. $\frac{9}{4}$

11. Simplify $f(\frac{x}{x+1})$

a. $\frac{(a-1)x^2 - 5x - 4}{(x+1)^2}$

b. $\frac{ax^2 + (2a-3)x + 1}{x^2}$

c. $\frac{(a-1)x^2 + 3x - 4}{(x+1)^2}$

d. $\frac{ax^2 - 5ax + 4}{(x+1)^2}$

e. $\frac{-ax^2 + 3x - 2}{(x+1)^2}$

12. There is one value of a for which $f(x)$ has a double root. Find the value of the double root.

a. $-\frac{9}{16}$ b. $\frac{9}{3}$ c. $\frac{8}{3}$ d. $-\frac{3}{4}$ e. $-\frac{8}{3}$

13. An odd function is one for which $f(-x) = -f(x)$. If $f(x)$ and $g(x)$ are odd functions, what must be true?

I. $(f+g)(x)$ must be odd.
II. $(fg)(x)$ must be odd.
III. $f(g(x))$ must be odd.

a. I only b. II only c. III only d. I and III e. II and III

14. Find all values of x for which

$$|3x+2| < |x-4|.$$

a. $-3 < x < \frac{1}{2}$ b. $x < \frac{1}{2}$ c. $x < -3$ or $x > \frac{1}{2}$ d. $x < -3$ e. $x > -3$

15. If $F(x) = Ax^2+Bx+C$ and $f(x) = ax^2+bx+c$ are quadratic functions with $F(x) \neq f(x)$, what is true about the number of solutions to $F(x) - f(x) = 0$?

I. It is possible that there is no solution.
II. It cannot have more than 2 solutions.
III. If it has only one solution then $A = a$.

a. II only b. I only c. I and II d. I and III e. All are true.

16. Sue always leaves a 17.5% tip when she dines out and **she calculates her tip based on the total bill including tax**. Last time she went out, she noticed that there was an extra sandwich on her bill and that the total was exactly \$20. She paid \$20 and told the server to take the sandwich off the bill and keep the change. If sales tax is 6.75%, how much (to the nearest penny) did the sandwich cost before tax?

a. \$3.42 b. \$3.28 c. \$2.98 d. \$2.68 e. \$2.79

17. Suppose that a, b and c are the roots of $f(x) = x^3 - 4x^2 - 5x$. Compute $(a+1)(b+1)(c+1)$.

a. -12 b. 5 c. 0 d. -6 e. 8

18. Jeff's Algebra I test average will be a 88 if he gets a 72 on the next exam and a 91 if he gets a 90 on the next exam. If all exams are weighted the same, how many exams has Jeff taken so far?

a. 3 b. 4 c. 5 d. 6 e. 7

19. Consider the parabola $y = x^2 - 3x + 5$. A line intersects this parabola at two points. One point is $(2, 3)$ and the other point is exactly $\sqrt{8}$ units away. Find the equation of this line.

a. $y = 3x - 3$ b. $y = x + 1$ c. $y = 5x - 7$ d. $y = -3x + 9$ e. $y = -x + 5$

20. The sum of Bob and Carol's ages is half of Alice's age. Carol will be as old as Bob is now in two years. In 6 years, Alice's age will be the sum of Carol's and Bob's ages. How old is Carol?

a. 2 b. 3 c. 4 d. 5 e. 6

21. Mary wants to withdraw \$200 from the bank in \$5, \$10, and \$20 bills. She wants to have at least one of each bill and would like the same number of \$5s as \$10s. What is the fewest number of bills she can receive?

a. 11 b. 12 c. 14 d. 15 e. 16

22. If $g(x) = \sqrt{2x + 3}$ find $g^{-1}(\frac{1}{4})$.

a. $\frac{2\sqrt{7}}{7}$ b. $\frac{1}{16}$ c. $\sqrt{11}$ d. $-\frac{47}{32}$ e. $\frac{\sqrt{11}}{11}$

23. The graphs of the equations

$$y = 3x^2 - 2x - 4 \qquad \text{and} \qquad y = -x^2 + 5x - 2$$

meet at two points. Find the point of intersection closest to $(0, 0)$.

a. $(-\frac{1}{4}, -3)$ b. $(0, -2)$ c. $(2, 4)$ d. $(-2, 12)$ e. $(-\frac{1}{4}, -\frac{53}{16})$

24. Jack's allowance is \$100 for the first month and then it increases by \$20 every month thereafter. Jill gets \$1 the first month and it doubles every month thereafter. Over the course of the year, who earned more total money and how much more did he/she earn?

a. Jill gets \$1575 more.
b. Jill gets \$1728 more.
c. Jack gets \$76 more.
d. Jack gets \$472 more.
e. They receive the exact same amount.

25. Frank and Jim decide to ride their bikes along a predetermined route. Frank can ride at a steady 18 mph and Jim can ride at 19.5 mph. When Jim finishes the ride he has to wait 20 minutes for Frank to finish. If they both rode the same distance, how far did each of them ride?

a. 30 mi. b. 78 mi. c. 4.5 mi. d. 36 mi. e. 62 mi.

26. Solve the following expression for b:

$$x = \frac{ab}{3a^2 - b^2}.$$

a. $\dfrac{ab - 3a^2x}{x}$

b. $\dfrac{ax}{3a^2 - x^2}$

c. $\dfrac{-a \pm |a|\sqrt{1 + 12x^2}}{2x}$

d. $\dfrac{-a \pm ax\sqrt{13}}{2x}$

e. $\dfrac{-x \pm \sqrt{x^2 - 4ax}}{2a}$

27. Expand $(3x - y^{-1})^{-3}$.

a. $\dfrac{y^3}{27x^3y^3 - 27x^2y^2 + 9xy - 1}$

b. $\dfrac{y^3}{27x^3 - 1}$

c. $\dfrac{-y^3}{27x^3}$

d. $\dfrac{1}{27x^3 - 9x^3y + 9xy^2 - y^3}$

e. $\dfrac{y^3}{27x^3y^3 - 9x^2y + 9xy - 1}$

28. Determine all values of c for which $3x^2 + 12x + c = 0$ has no (real) solutions.

a. $c < 0$ b. $c \leq 0$ c. $c \leq 12$ d. $c > 12$ e. $0 < c < 12$

29. Suppose a linear function, $f(x)$, contains the points $(1, -7)$ and $(3, 4)$. What is the change in $f(x)$ when x increases by 3?

a. 9 b. $\dfrac{33}{2}$ c. $-\dfrac{9}{2}$ d. -9 e. 33

30. A custom T-shirt shop charges different prices per shirt based on how many the customer wants. For up to 10 shirts, the cost is \$10 per shirt. When ordering between 11 and 24 (inclusive) shirts, the cost is only \$9 per shirt. For 25 to 50 shirts the cost per shirt is \$8; and the cost per shirt is \$7 for any order of more than 50 shirts. Alex determines that there is a number of shirts x so that ordering x shirts and ordering $x + 3$ shirts costs exactly the same. What is x?

a. 8 b. 9 c. 23 d. 24 e. 49

31. When the polynomial $P(x)$ is divided by $x-3$ the quotient is $2x^2-4x+3$ and the remainder is 2. Evaluate $P(2)$.

a.-1 b. 3 c. 1 d. 5 e. cannot be determined

32. Suppose p and q are distinct primes. Which of the following statements are true?
I. $p+q$ is never prime.
II. pq has exactly 4 positive integer factors.
III. The least common multiple of p and q is 1.

a.I only b. II only c. III only d. I and II e. II and III

33. A line segment connects the points $(1,7)$ and $(22,42)$ in the plane. How many points (including the endpoints) on this segment have the property that both coordinates are integers?

a.9 b. 22 c. 21 d. 7 e. 8

34. Five consecutive integers have the property that the sum of the first 4 is exactly three times the fifth. Find the sum of the next five consecutive integers.

a.80 b. 65 c. 55 d. 90 e. 75

35. Find the digit x so that the number $111111x11111111$ is divisible by 9.

a.2 b. 3 c. 4 d. 5 e. 6

36. Let $a \star b$ be defined by $a \star b = \frac{1}{a} + \frac{1}{b}$. Find the solution set for $x \star (x \star 1) = 2$.

a.$\{\frac{1 \pm \sqrt{5}}{2}\}$ b. $\{-1, 2\}$ c. $\{-2, -1\}$ d. $\{-2, \frac{1}{2}\}$ e. $\{\frac{-1 \pm \sqrt{5}}{2}\}$

37. Let $P(x)$ be a polynomial with degree 5. What is the greatest number of times a line $y = mx + b$ can intersect $P(x)$?

a.1 b. 4 c. 5 d. 6 e. Any number is possible.

38. Let $M(x, y)$ denote the larger of x and y and $m(x, y)$ denote the smaller of x and y. Suppose that $a < b < c < d < e$. Find

$$M\left(m\left(M\left(a,b\right),c\right),m\left(d,M\left(c,e\right)\right)\right).$$

a.a b. b c. c d. d e. e

39. Today is a Thursday. What day of the week will it be 2008 days from now?

a.Thursday b. Friday c. Sunday d. Monday e. Wednesday

40. A fast food chain wants to advertise that their burgers can be customized in over a million ways. What is the smallest number of toppings they need to provide to make this statement true?

a.20 b. 26 c. 51 d. 8 e. 17