Algebra I

State Mathematics Contest Finals, May 3, 2007

1. Solve for x, given that $3x + 5 = 5(x + 3)$.

   a. –1 b. 1 c. –3 d. 3 e. –5

2. If today is Thursday then what day of the week will it be 1,000 days from now?

   a. Monday b. Wednesday c. Friday d. Saturday e. Sunday

3. Think of a 2-digit number, and call it "N". Double it, and add 12. Then divide that sum by two and subtract your original number, N. What is the result?

   a. 6 b. N c. 12 d. $N + 6$ e. none of these

4. Let n be the smallest natural number with the properties that when divided by 11 it has a remainder of 4 but when divided by 13 it has a remainder of 7. What is the sum of the digits of this number?

   a. 7 b. 9 c. 11 d. 14 e. none of these

5. If the points (1, 5), (4, 13), and (9 , c) are colinear, what is the value of c?

   a. $13\frac{1}{3}$ b. 18 c. 21 d. $26\frac{1}{3}$ e. $29\frac{7}{13}$

6. A knight wanted to visit a princess. If he traveled at 15 km per hour, he would arrive one hour too early. If he traveled at 10 km per hour, he would arrive one hour too late. What is the distance the knight must travel?

   a. 30 km b. 37.5 km c. 60.0 km d. 75.0 km e. none of these.

7. $\dfrac{7-\sqrt{5}}{3+\sqrt{5}}$ can be rewritten as $a+b\sqrt{5}$ where a and b are rational numbers. Calculate ab.

a. 4 b. $-\dfrac{52}{5}$ c. $-\dfrac{65}{4}$ d. $-\dfrac{75}{4}$ e. –260

8. Letters have been substituted for digits. Each letter represents a unique digit, different letters represent different digits. If "TO + GO = OUT" then what is "GOT–TUG"?

a. 514 b. 604 c. 614 d. 714 e. none of these

9. Given the parabola $y = 4 - x^2$ and the line $y = c - x$. For which value of c will the two graphs have only a single point in common?

a. 2 b. 4 c. $\sqrt{17}$ d. 4.25 e. 17

10. If $x^2 + y^2 = 17$ and $x + y = 4$, what does xy equal?

a. $-\frac{1}{2}$ b. $\frac{1}{2}$ c. –1 d. 8 c. none of these

11. Arnold earned a 78 on his essay, had a 98 on his mid term, but his class participation grade was a 70. If the essay and mid term are each worth 20% of his grade, and class participation is 30%, what grade must he make on his final exam to get an average of 85?

a. 85 b. $93.\overline{3}$ c. 94 d. 96 e. 100

12. Solve for n in terms of y given $y = \dfrac{2n+1}{n-1}$.

a. $n = \dfrac{y-1}{y+2}$ b. $n = \dfrac{y+1}{2y-1}$ c. $n = \dfrac{y-1}{2y+1}$ d. $n = \dfrac{2y+1}{y-1}$ e. $n = \dfrac{y+1}{y-2}$

13. Divide the expression 3^{2007} by 7 and determine the remainder.

a. 6 b. 5 c. 4 d. 3 e. 2

14. A lily pad doubles in size each day. If it takes 28 days for the lily pad to cover the entire pond, how many days will it take for it to cover one eighth of the pond?

a. 3.5 b. 7 c. 14 d. 20 e. 25

15. Barry, Carol, and Donald went to eat at a fancy restaurant. They each paid for their own meals. Barry left a 15% tip, Carol left a 20% tip, and Donald left $1. The resulting total tip was $6.30. If Barry's meal was $5 more than Carol's, but Donald's meal cost as much as Carol's and Barry's combined, how much was Donald's meal?

a. $13.00 b. $18.00 c. $28.30 d. $31.00 e. $32.28

16. If $x + 2y = 12$ and $x - y = 3$ find the value of $x + y$.

a. 3 b. 5 c. 9 d. 17 e. none of these

17. Let "*abc*" be a three digit number where *a*, b, and *c* are the digits. If the sum of the digits is 10, $0 < a < b < c$, and $3a + 2b + c = 17$ find the value of $a^2 + b^2 + c^2$.

a. 38 b. 42 c. 46 d. 52 e. 58

18. You can convert temperatures from Celsius (C) to Fahrenheit (F) using following formula:

$$F = (9/5C) + 32.$$

At what temperature is the Fahrenheit temperature equal to the Celsius temperature?

a. -40° b. 0° c. 32° d. 160° e. 212°

19. Find the sum of all the solutions to the equation: $|3x+1|-3=x+12$.

a. 7 b. 3 c. –4 d. –7 e. none of these

20. Given $f(x)=\dfrac{2x+1}{x-2}$, solve for a where $f(f(a))=1$.

a. –3 b. –1 c. $-\dfrac{1}{2}$ d. 1 e. $\dfrac{1+\sqrt{5}}{2}$

21. Which of these statements is false for some natural number n?

a. $n^2 + 1$ is never divisible 3
b. $n^2 + 1$ is never divisible 4
c. $n^2 + 1$ is never divisible 7
d. $n^2 + 1$ is never divisible 11
e. $n^2 + 1$ is never divisible 13

22. Given $(x-y)^2 = x^2 - xy - y^2$ where $x \neq 0$, find the solution set for $\dfrac{y}{x}$.

a. $\{0, 2\}$ b. $\{\pm\sqrt{2}\}$ c. $\{1\pm\sqrt{2}\}$ d. $\left\{0, \dfrac{1}{2}\right\}$ e. $\left\{\dfrac{1}{2}, 2\right\}$

23. Find the x intercept of the line that passes through the points (0, 51) and (5, 36).

a. 17 b. 18.2 c. $19\frac{1}{3}$ d. 21 e. 51

24. Let "*abc*" be a 3-digit number for which $a > c > 0$, and let $x = abc - cba$. Which of these is NOT necessarily true for x?

a. $x < 800$ b. $x > 99$ c. The ten's digit is 9 d. 9 divides x e. 11 divides x

25. If $2x + 3y - z = 7$ and $3x + 2y + 6z = 8$, determine $x + y + z$.

a. 15 b. 5 c. 3 d. –1 e. can not be determined

26. Suppose you have a bowl with 9 chips numbered 1 through 9 and assume you take out two chips, what is the chance that the sum of the values will be an odd number?

a. $\frac{4}{9}$ b. $\frac{1}{2}$ c. $\frac{19}{36}$ d. $\frac{5}{9}$ e. $\frac{2}{3}$

27. The Croatian basketball team played the Estonian national team. If Estonia lost by 3 points, and total number of points scored in this game was 117, what was Croatia's score?

a. 57 b. 59 c. 60 d. 63 e. none of these

28. Assume you have 14 silver-colored coins in your pocket (nickels, dimes, and quarters), and the value of the quarters is equal to the value of the remaining coins. If you have at least one coin of each denomination, how many nickels do you have?

a. 3 b. 4 c. 6 d. 8 e. none of these

29. If $2^x = 15$ then determine the value of $\sqrt{0.6 \cdot 8^x}$.

a. $15\sqrt{2}$ b. $9\sqrt{15}$ c. 45 d. $30\sqrt{3}$ e. 75

30. A man spends one fifth of the money in his wallet. He then spends one fifth of what remains in the wallet. If he spent \$36.00 in all, how much money did he have to begin with?

a. \$75 b. \$100 c. \$120 d. \$150 e. \$180

31. In North Carolina a typical license plate has three letters followed by four numbers. Approximately how many different license plates can be issued using this convention?

a. 1.76×10^{8} b. 4.57×10^{8} c. 1.76×10^{9} d. 6.33×10^{9} e. 7.83×10^{10}

32. Given the function $f(x) = 2x + 1$, solve for x where $f(3x + 7) = 13$.

a. $-\frac{1}{3}$ b. $\frac{2}{3}$ c. 2 d. 10 e. none of these

33. The line $y = mx + 1$ intersects the circle, $x^2 + y^2 = 1$ at two points. One of these points is (0, 1) find the y-coordinate of the other point.

a. $m + 1$ b. $\frac{-2m}{1+m^2}$ c. $\frac{1-m}{\sqrt{1+m^2}}$ d. $\frac{1-m^2}{1+m^2}$ e. $\frac{\sqrt{m^2+2m}}{m^2+1}$

34. Chris and Pat both had the same amount of money in their savings accounts. It was a very nice bank that paid 6% interest compounded annually. At the beginning of the year Chris withdrew half of his savings while Pat deposited $200 into her account. A year later Chris deposited $200 into his account while Pat withdrew an amount equal to the amount that Chris had taken out of his account the year before. Afterward Pat's account still had $300 more than Chris's. How much was the original amount in each of their accounts?

a. Less than $9,000 b. $9,000 to $9,499 c. $9,500 to $9,745

d. $9,750 to $9,999 e. More than $10,000

35. Two candles, #1 and #2, of the same length are lit at the same time. Candle #1 will burn for six hours while candle #2 burns down twice as fast. How long must the two candles burn before the candle #1 is three times as long as candle #2?

a. 2 hrs. b. 2 hrs. 12 min. c. 2 hrs. 24 min. d. 2 hrs. 40 min. e. 3 hrs.

36. Which value approximates $2^{1,000}$?

a. 10^{200} b. 10^{301} c. 10^{693} d. 10^{602} e. 10^{500}

37. Let a be a real number. What is the coefficient of the x^3 term of the expansion: $(x + 2a)^5$?

a. $4a$ b. $10a^2$ c. $20a^2$ d. $40a^2$ e. $80a^3$

38. If n is a natural number, then n^3 can be written as a sum of n consecutive odd integers.
For example, if $n = 3$, then $3^3 = 7 + 9 + 11$.
What is the expression in terms of n of the first odd integer in the sequence that will yield n^3?

a. $2n + 1$ b. $3n - 2$ c. $n^2 - 2$ d. $n^2 + n - 5$ e. $n^2 - n + 1$

39. A father promises to pay his daughter \$5.00 for each correct answer she obtains on the math contest, but she must pay \$3 for every answer that is not correct. If the father ended up paying the daughter \$16 how many did she get correct on this 40 question test?

a. 13 b. 15 c. 17 d. 18 e. none of these

40. Which of the following equations does NOT have as a solution the positive number with the property that if you subtract its multiplicative inverse from itself you get 1?

a. $\frac{x^2-1}{x}=1$ b. $\frac{1}{x-1}=x$ c. $x^2=x+1$ d. $x^3=2x+1$ e. $x=\frac{1-x}{x}$