GEOMETRY
State Mathematics Finals Contest, May 4, 2006

1. Which statement does not guarantee that quadrilateral *WXYZ* is a parallelogram?
   a. $\overline{WX} \cong \overline{YZ}; \overline{XY} \parallel \overline{WZ}$.
   b. $\angle W \cong \angle Y; \angle X \cong \angle Z$.
   c. $\overline{WX} \cong \overline{YZ}; \overline{XY} \cong \overline{WZ}$.
   d. $\overline{XY} \parallel \overline{WZ}; \overline{WX} \parallel \overline{ZY}$.
   e. $\overline{XY} \cong \overline{WZ}; \overline{XY} \parallel \overline{WZ}$.

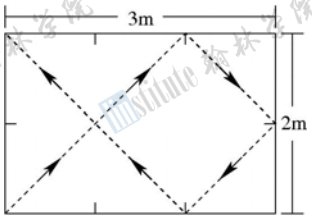
2. Mad scientist Dr. I B. Goode finds a clump of mysterious metal on his laboratory floor. He weighs it and finds that the clump weighs 685.8 grams. He then drops it into a cylindrical container, causing the water level to rise 1.3 cm. The radius of the base of the container is 4.0 cm. Use the table below to determine the type of metal (assuming it is pure).

Metal	Silver	Lead	Platinum	Gold
Density g/cm^3	10.5	11.3	21.4	19.3

   a. Silver.
   b. Lead.
   c. Platinum.
   d. Gold.
   e. None of these.

3. Assume that a pool ball is the size of a point and that the only pockets of the pool table are located exactly in its corners. When the pool table is 2 m by 3 m and the ball is shot at a 45° angle as shown, it bounces three times and then goes into the upper left corner. If we shoot a ball at a 45° angle from the lower left corner of a giant pool table that is 165 m by 297 m, how many times will it bounce before it goes into a pocket and into which corner does it eventually go? (Assume 165 m is the vertical dimension)

   a. 12 bounces, upper right corner.
   b. 10 bounces, lower right corner.
   c. 12 bounces, upper left corner.
   d. 8 bounces, upper left corner
   e. 8 bounces, lower right corner.



4. Consider an experiment using 2 straws of unequal length. In the experiment, you are asked to find the midpoints of the straws, then make an X so the straws intersect at their midpoints. Finally draw the figure you get by joining the endpoints. What kind of quadrilateral might you get?

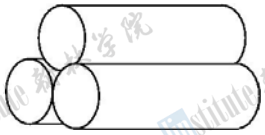
   a. Rectangle.
   b. Trapezoid.
   c. Rhombus.
   d. Square.
   e. None of these.

5. Consider the conditional statement "if the diagonals of a quadrilateral are perpendicular to each other, then the quadrilateral is a rhombus." Which of the following is true?

   a. Only the conditional statement.
   b. Only the converse of the conditional statement.
   c. Only the inverse of the conditional statement.
   d. The conditional statement, its converse and its inverse.
   e. The converse and the inverse of the conditional statement, but not the conditional statement itself.

6. Three cylinders are placed on a table, as shown in the diagram below. Each cylinder has a diameter of 150 centimeters. To the nearest centimeter, how tall is the stack?

   a. 275 cm.
   b. 280 cm.
   c. 290 cm.
   d. 295 cm.
   e. 300 cm.



7. Which of the following triples could not be the sides of a triangle?
   a. 7, 7, 8.
   b. 3, 4, 6.
   c. 2, 7, 11.
   d. 17, 18, 25.
   e. 1023, 2168, 3040.

8. Let *ABC* be an equilateral triangle with sides *x*. Let *P* be the point of intersection of the three angle bisectors. Find $\overline{AP}$.

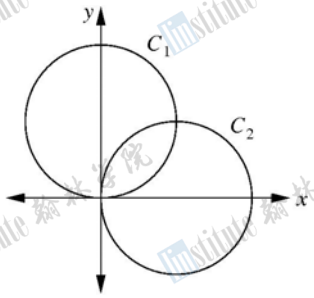
   a. $\frac{x\sqrt{3}}{3}$.
   b. $\frac{x\sqrt{3}}{6}$.
   c. $\frac{x\sqrt{3}}{4}$.
   d. $\frac{5x\sqrt{3}}{6}$.
   e. $\frac{2x\sqrt{3}}{3}$.

9. $\overrightarrow{OB}$ bisects $\angle AOC$. If $m\angle AOB = 3x + 16$ and $m\angle BOC = 8x - 14$, then $m\angle AOC =$

   a. 6.
   b. 20.
   c. 34.
   d. 56.
   e. 68.

10. Circle C_1 has center (0,2) with radius 2, and circle C_2 has center (2,0) with radius 2. The circles overlap in the first quadrant. What is the area of overlap?

a. $3\pi - 7$ square units.
b. $2\pi - 4$ square units.
c. $\pi - 1$ square units.
d. $2\pi - 2$ square units.
e. $4 - \frac{\pi}{2}$ square units.

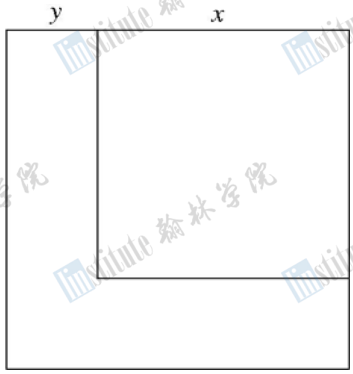


11. Two cylindrical water tanks stand side by side. One has a radius of 4 meters and contains water to a depth of 12.5 meters. The other has a radius of 3 meters and is empty. Water is pumped from the first tank to the second tank at a rate of 10 cubic meters per minute. How long, to the nearest tenth of a minute, must the pump run before the depth of the water is the same in both tanks?

a. 18.8.
b. 19.8.
c. 20.4
d. 21.6
e. 22.6

12. A large square has a smaller square cut from its corner in such a way that the area of the square removed equals the area of the remaining region. If *x* represents the length of a side of the removed square, and *y* represents the remaining length, find the ratio $\frac{x}{y}$.

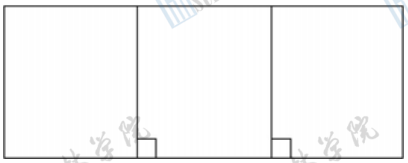
a. $\frac{5}{2}$.
b. $\frac{12}{5}$.
c. $\frac{1+\sqrt{2}}{1}$.
d. $\frac{8}{3}$.
e. $\frac{6(\sqrt{2}-1)}{1}$.



13. A chord which is the perpendicular bisector of a radius of length 12 in a circle, has length:

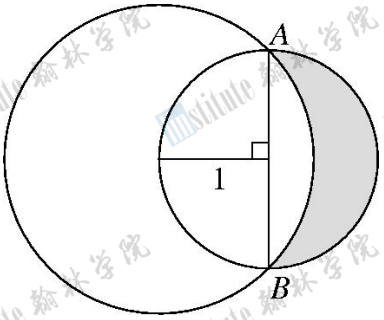
a. $3\sqrt{3}$.
b. 27
c. $6\sqrt{3}$.
d. $12\sqrt{3}$.
e. None of these.

14. A farmer has sixty meters of fence with which to build a rectangular animal run as shown for her cows, horses, and pigs. She wants each type of animal to have the same area. What is the largest number of square meters that can be enclosed?



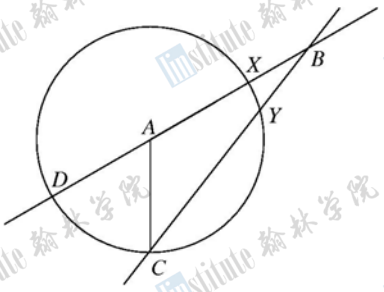
a. 112.5.
b. 124.
c. 128.5.
d. 135.
e. 136.5.

15. The center of a circle lies on a unit circle and passes through the endpoints of a diameter $\overline{AB}$. Find the area of the crescent indicated by the shaded region of the diagram



a. $\left|\frac{1}{2}\pi - 1\right|$.
b. $\frac{1}{2}\pi + 1$.
c. $\frac{1}{2}\pi$.
d. 1.
e. None of these.

16. Given the circle with center A. If $m\angle DAC = 70^\circ$ and $m\angle ABC = 20^\circ$. Find the measure of the arc XY.



a. 20°.
b. 30°
c. 40°.
d. 70°.
e. None of these.

17. An inflated round balloon with radius r centimeters holds approximately 523,600 cm^3 of air. When the balloon in contracted such that the radius is $\frac{2}{3}$ the original size, the approximate volume of the partially inflated balloon is

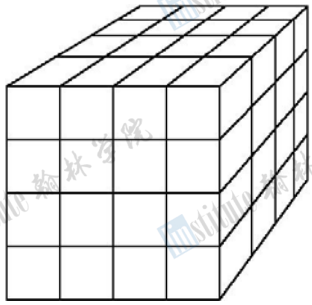
a. $1.55\times10^5\ \text{cm}^3$.
b. $1.75\times10^5\ \text{cm}^3$.
c. $3.41\times10^5\ \text{cm}^3$.
d. $3.49\times10^5\ \text{cm}^3$.
e. $4.39\times10^5\ \text{cm}^3$.

18. A triangle is inscribed in a circle of radius 4 units such that one side of the triangle is a diameter of the circle and one angle of the triangle has a measure of 30°. If all regions in the interior of the circle but not in the interior of the triangle are shaded, what is the approximate area of the shaded regions?

    a. 33.33 square units.
    b. 33.70 square units.
    c. 33.81 square units.
    d. 34.26 square units.
    e. 36.41 square units.

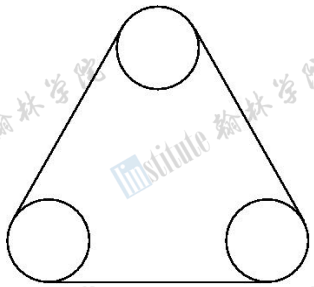
19. A cubical block of wood is painted blue and sliced into n^3 smaller blocks of equal size. The figure shows how this is done for $n = 4$. Find a general formula for the number of smaller blocks with exactly one blue face when $n > 4$.

    a. $(n-1)^3$.
    b. $12(n-2)$.
    c. $6n$.
    d. $6(n-2)^2$.
    e. $6(n^2-4)$.



20. Three wheels, each of radius 1, have their centers at respective vertices of an equilateral triangle of side length 4. A belt is wrapped continuously around the wheels. Find the length of the belt.

    a. $3(\pi+4)$.
    b. $\frac{2\pi}{3}+4$.
    c. $2(\pi+6)$.
    d. $\frac{3\pi}{2}+12$.
    e. None of these.



21. A rectangular piece of paper is folded in half as shown in figure I below. If the two opposite corners of the folded paper are cut off as shown in figure II, which of the following is the design of the paper when unfolded? **No folding is permitted during testing.**

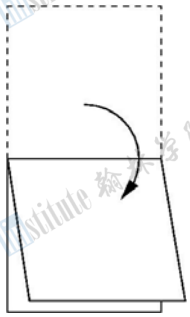


Figure I

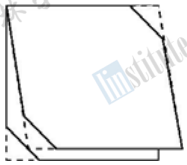
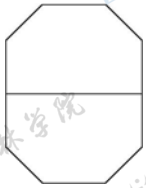
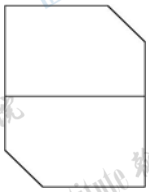


Figure II

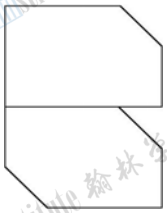
a.



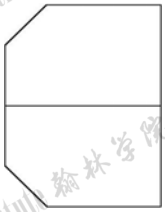
b.



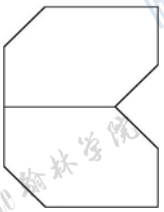
c.



d.



e.



22. Mr. Goebel keeps his goat on a 3 meter chain connected to a metal hook in the ground. Sometimes he chains the goat to a metal hook in the center of his big yard. At other times the chain is attached at ground level to the center of the 4 meter wall of his 4 meter by 5 meter shed. How much more grass can the goat reach to eat when the metal hook is located in the center of the yard instead of attached to the center of the 4 meter wall of the shed?

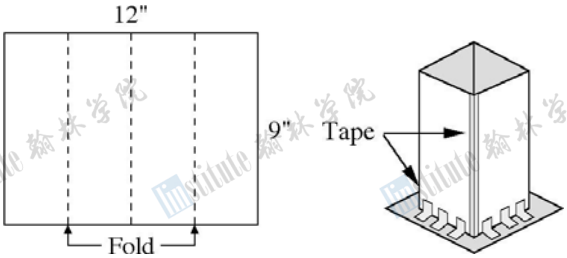
    a. $11.68\,m^2$.
    b. $12.57\,m^2$.
    c. $13.32\,m^2$.
    d. $14.84\,m^2$.
    e. $15.71\,m^2$

23. Five women have lunch together seated around a circular table. Ms. Osborne is sitting between Ms. Lewis and Ms. Martin. Ellen is sitting between Cathy and Ms. Norris. Ms. Lewis is between Ellen and Alice. Cathy and Doris are sisters. Betty is seated with Ms. Parkes on her left and Ms. Martin on her right. Which of the following is not true?

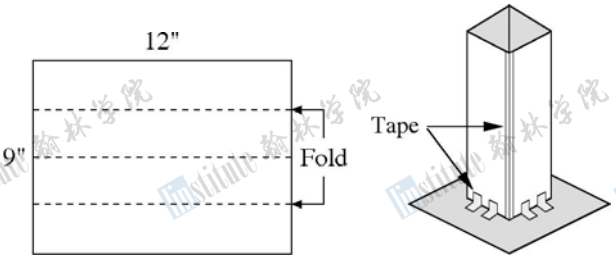
    a. Alice Osborne sat next to Cathy Lewis
    b. Doris Martin sat next to Betty Norris.
    c. Someone sat between Ellen Parkes and Alice Osborne
    d. Alice sat next to Betty.
    e. Cathy sat between Ellen and Alice.

24. Two students were asked to construct "tubes" and then fill them to determine which had the greater volume. Each tube was constructed from a 9" × 12" piece of construction paper. For the short tube, Jack folded the paper as shown in **A** and for the tall tube, Bill folded the paper as shown in **B**. Which tube has the greater volume and how much greater?

A



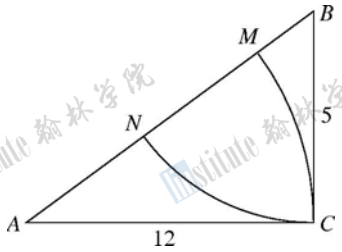
B



a. Neither. The tubes have the same volume.
b. The short tube has 20.25 cubic inches more volume than the tall tube.
c. The short tube has 19.75 cubic inches more volume than the tall tube.
d. The tall tube has 17.95 cubic inches more volume than the short tube.
e. The tall tube has 20.25 cubic inches more volume than the short tube.

25. In right triangle ABC with legs 5 and 12, arcs of circles are drawn, one with center A and radius 12, the other with center B and radius 5. They intersect the hypotenuse in M and N. Then $\overline{MN}$ has length

a. 2.
b. 2.6.
c. 3.
d. 4.
e. 4.8.



26. Let C_1 and C_2 be circles of radius 1 that are in the same plane and tangent to each other. How many circles of radius 3 are in this plane and tangent to both C_1 and C_2?

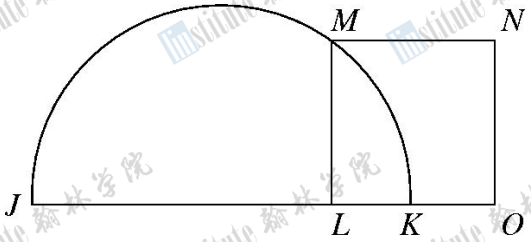
a. 2.
b. 4.
c. 5.
d. 6.
e. 8.

27. Seven different points are marked on the circumference of a circle. How many different triangles can be formed by connecting these points?

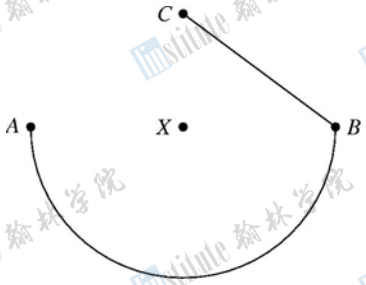
a. 7.
b. 14.
c. 21.
d. 22.
e. 35.

28. *JMK* is a semicircle, $JL = 10$ units and $LK = 2$ units. What is the area of the square *LMNO*?

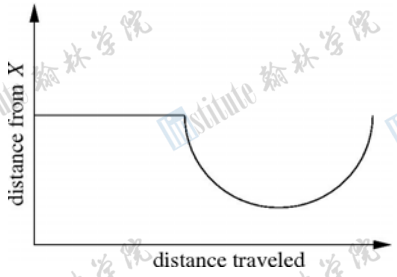
a. $2\sqrt{13}$ units^2.
b. $4\sqrt{5}$ units^2.
c. 16 units^2.
d. 20 units^2.
e. 24 units^2.



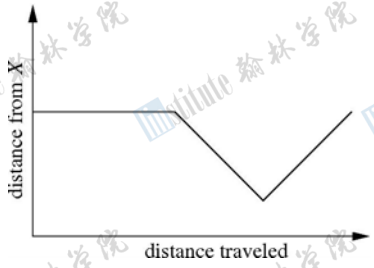
29. A ship travels from point *A* to point *B* along a semicircular path, centered at Island *X*. Then it travels along a straight path from *B* to *C*. Sketch a graph that best shows the ship's distance from Island *X* as it moves along its course.



a.

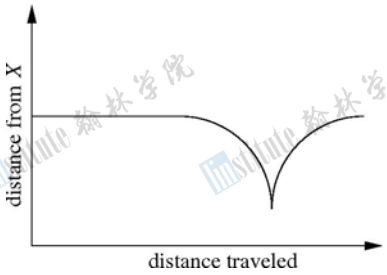


b.

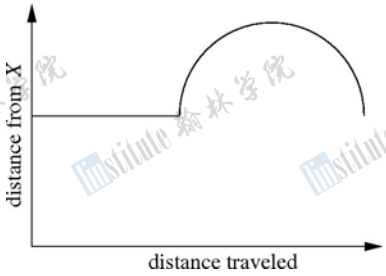


c.

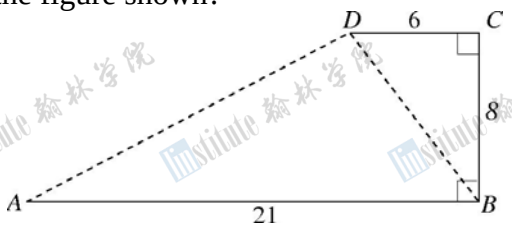
d.



e.



30. What is the sum of the distances *AD* and *BD* in the figure shown?



a. 21.
b. 23.
c. 25.
d. 27.
e. 29.

31. Let triangle *ABC* be any non-equilateral triangle of area 1. For which points *P* can the area of triangle *PAB* be determined?

a. *P* is the centroid of triangle *ABC*.
b. *P* is the incenter of triangle *ABC*.
c. *P* is the circumcenter of triangle *ABC*.
d. More than one of the above.
e. None of the above.

32. A metal flagpole on level ground broke at a certain point with the top part tipping over like a hinge and the tip hitting the ground at a point 20 ft from the base. It was rewelded but again broke, this time at a point 5 ft lower than before, with the tip hitting the ground at a point 30 ft from the base. The height of the flagpole is:

a. 21 ft.
b. 29 ft.
c. 50 ft.
d. 53 ft.
e. 47.5 ft.

33. On a standard clock, at what time between 4:00 and 5:00 do the minute hand and the hour hand coincide?

a. 4:21:49.
b. 4:23:15.
c. 4:24:36.
d. 4:25:18.
e. 4:26:56.

34. A square-shaped building is oriented in a north-south and east-west direction. The walls are 200 ft on each edge with a door in the center of each wall. A tree is located 15 ft from the center of the east door in a due east direction. How far from the south door must someone walk due south in order to see the tree?

a. $\frac{2000}{3}$ ft.
b. $\frac{2000}{4}$ ft..
c. $\frac{2000}{5}$ ft.
d. $\frac{2000}{6}$ ft.
e. None of these.

35. Three parallel lines ℓ_1, ℓ_2 and ℓ_3 are drawn through the vertices *A*, *B*, and *C*, respectively, of a square *ABCD*. If the distance between ℓ_1 and ℓ_2 is 7 and between ℓ_2 and ℓ_3 is 12, find the area of *ABCD*.

a. 169 square units.
b. 182 square units.
c. 192 square units.
d. 193 square units.
e. 200 square units.

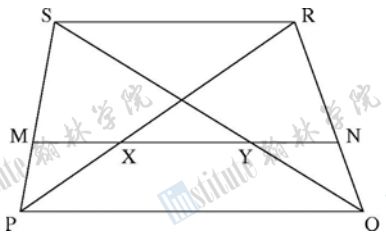
36. If $\overleftrightarrow{AB}$ intersects $\overleftrightarrow{CD}$ at *E*, which word or words can be used to describe $\angle AEC$ and $\angle BEC$?

I. supplementary.
II. congruent.
III. adjacent

a. I only.
b. I, II only.
c. II, III only.
d. I, III only.
e. I, II, III.

37. *PORS* is a trapezoid with $\overline{PO}$ a base. Median $\overline{MN}$ intersects the diagonals at *X* and *Y*. If $SR = 12$ and $XY = 3$, find *PO*.

a. 15.
b. 16.
c. 18.
d. 21.
e. 24.



38. A regular hexagon is formed by joining successively the midpoints of the sides of a regular hexagon. The ratio of the area of the smaller hexagon to the area of the larger hexagon is:

    a. $\frac{2}{3}$.
    b. $\frac{3}{4}$.
    c. $\frac{4}{5}$.
    d. $\frac{\sqrt{6}}{3}$.
    e. $\frac{\sqrt{3}}{2}$.

39. What is the ratio of the area of a square that circumscribes a circle to the area of a square that inscribes the same circle?

    a. $2:1$.
    b. $3:2$.
    c. $2:\sqrt{2}$.
    d. $\sqrt{3}:2$.
    e. $5:3$.

40. From a hot-air balloon 2 km high, the angles of depression to two towns east of the balloon and in line with the balloon are 81.2° and 13.5°. Which of the following best approximates the distance between the two towns?

    a. 7.5 km.
    b. 8 km.
    c. 8.5 km.
    d. 9 km.
    e. 9.5 km.

ANSWER KEY
2006 Geometry State Finals

1. A	11. E	21. E	31. A
2. A	12. C	22. B	32. C
3. A	13. D	23. D	33. A
4. C	14. A	24. B	34. A
5. E	15. D	25. D	35. D
6. B	16. B	26. D	36. D
7. C	17. A	27. E	37. C
8. A	18. E	28. D	38. B
9. E	19. D	29. A	39. A
10. B	20. C	30. D	40. B

Tie Breakers:
Best of Three: 3, 20, 26
Sudden Death: 6, 11, 15, 33, 37