Polynomials

Solutions:

1. D. $2x+c=6 \Rightarrow 2x=6-c \Rightarrow x=3-\frac{c}{2}$

2. C. The cost of the iPod is $\$300 \cdot 1.07 = \$321$, so he must save an additional \$210, which, at \$5 per hour, will take $\frac{\$210}{\$5 \text{ per hour}} = 42$ hours.

3. A. $\frac{2}{A+1} - \frac{1}{A-1} + \frac{2A}{A^2-1} = \frac{2(A-1)}{A^2-1} - \frac{A+1}{A^2-1} + \frac{2A}{A^2-1} = \frac{2A-2-A-1+2A}{A^2-1} = \frac{3A-3}{A^2-1} = \frac{3(A-1)}{(A+1)(A-1)} = \frac{3}{A+1}$

4. E. $0,1,5,14,30,55,\ldots,91$. The terms are the sum of squares: $0 = 0^2, 1 = 0^2+1^2, 5 = 0^2+1^2+2^2 = 5$, etc. The next term is 6^2 more than 55, and so 91.

5. E. $f(x)$ has x-intercepts at $-1,3$, so $(x+1)$ and $(x-3)$ are factors of $f(x)$. E is the only solution that satisfies these criteria.

6. B. $x+y=17$, so $x^2+2xy+y^2=(x+y)^2=17^2=289$, so $2xy = 289-167=122$ and $xy=61$.

7. D. 3 divides x, 5 divides $(x+1)$ and 7 divides $(x+2)$. The first two properties limit the possibilities to $15k+9$. 9, 24, 39 fail the third property, but 54 satisfies it.

8. C. $\frac{16^k+16}{16} = 9 \Rightarrow 16^k = 128 \Rightarrow 2^{4k} = 2^7 \Rightarrow 4k = 7 \Rightarrow k = \frac{4}{7}$

9. A. $3+7+11+15+19+23+\cdots+399 = 201(100) = 20,100$ - the average value of each term is 201 and there are 100 terms.

10. E. To not have an intersection, two lines must be parallel, and so have the same slope, so $\frac{-12}{c} = \frac{5}{2}$ and $c = \frac{-24}{5} = -4.8$

11. C. The minute hand moves at 6° per minute and the hour hand moves at 0.5° per minute. The difference in rates is 5.5° per minute. To traverse the relative 240° until the next 120° position, takes $\frac{120}{5.5} = 43\frac{7}{11}$ minutes

12. A. There are 4 orders in which this could occur and the probability of each order is $0.7^3(1-0.7)$. Multiplying the probability by the orders gives 0.4116.

13. C. $\sqrt{x} = \sqrt[3]{9}$, so $x^2 = 9^{4/3} = 9\sqrt[3]{9}$

14. A. Let x be the distance outside of town when they meet, and let t be the amount of time that Beth spent driving (in hours)then $x = 70(t-\frac{1}{4}) = 60t$. $10t = \frac{70}{4}$, $t = \frac{7}{4}$. Substituting for t gives $60 \cdot \frac{7}{4} = 105$ mi.

15. B. $y = 3x - x^2$ intersects $y = kx+3$, so $3x - x^2 = kx+3$ and $x^2 + (k-3)x + 3 = 0$. The roots are then $\frac{3-k\pm\sqrt{k^2-6k+9-12}}{2}$. In order for this to be a single repeated root, $k^2 - 6k - 3 = 0$ and $k = \frac{6\pm\sqrt{36+12}}{2} = 3\pm\sqrt{12}$

16. E. AMMA x XA = AAAAAA. If A x A produces A as the last digit, then A is one of 1,5,6. To produce an A in the 10's digit, if A = 1, (M+X)=11, while if A = 5 or 6 then (M+X) = 13. If A = 1, X = 8, M = 3, which doesn't work, while if X = 9, M = 2 which does. If A = 5 or 6, none of the pairs of X,M that produce X+M = 13 produce a large enough product.

17. D. Let t be the current work time, w her former wage, then $2wt = 3w(t-12)$ and $t = 36$.

18. E. $((x-2)\star(x+4))\star(x+7) = \dfrac{\frac{(x-2)+(x+4)}{2} + (x+7)}{2}$
$= \dfrac{(x+1)+(x+7)}{2} = x+4$

19. E. $((x-c)\star x)\star(x+c) = x\star 2c, \dfrac{\frac{(x-c)+(x)}{2} + (x+c)}{2} = \dfrac{x+2c}{2}$
$x + \dfrac{c}{4} = \dfrac{x}{2} + c, x = \dfrac{3c}{2}$

20. A. If Fred breaks even at 12 cameras, then his purchase price per camera is $\dfrac{\text{initial cost + cameras x transaction cost}}{\text{cameras x (100\% + 50\% markup)}}$
$\frac{(4284+12\cdot21)}{12\cdot1.5} = \frac{4536}{18} = 252$. This gives 17 cameras. Subtracting the initial cost of \$4,284 and the transaction cost of $17\cdot\$21$ from 150% of \$4,284 gives a profit of \$1,785.

21. B. $x + \frac{1}{x} = \frac{1}{p}, x^2+1 = \frac{x}{p}, x^2 - \frac{x}{p} + 1 = 0, x = \dfrac{\frac{1}{p}\pm\sqrt{\frac{1}{p^2}-4}}{2} = \dfrac{1\pm\sqrt{1-4p^2}}{2p}$

22. B. $\dfrac{x^2-4}{x-4}$ is 0 or undefined when $x = -2, 2, 4$. Testing points in each of the intervals (e.x. $x = -3, 0, 3, 5$) gives $\{x|(x\le -2)\cup(2\le x<4)\}$

23. A. $\sqrt{x^2+(2x)^2}=5 \Rightarrow x=\sqrt{5}$

24. A. The smallest triangular number above 100 is 105, where $n=14$ and the smallest triangular number above 200 is 210, where $n=20$, so there are 6 triangular numbers in this range.

25. C. On successive levels there are $66+55+45+36+28+21+15+10+6+3+1=286$

26. E. This is calculated as $\binom{8}{2}=28$

27. E. $\frac{\sqrt{5}}{\sqrt{x}-\sqrt{5}}+\frac{\sqrt{x}}{\sqrt{x}+\sqrt{5}}=\frac{5+\sqrt{5x}+x-\sqrt{5x}}{x-5}=\frac{x+5}{x-5}$

28. C. $4^{2x+1}+8=33(4^x)$, $4^{2(1.5)+1}+8=33(4^{1.5})$, $4^{2(-1)+1}+8=33(4^{-1})$, so $1.5+-1=.5$

29. D. The least common denominator of 10 and 28 is 140.

30. A. $(231+122)x21=1013x21=1013+20320=21333$

31. D. The average is $65\cdot.2+82\cdot.3+93\cdot.2+x\cdot.3=85$, $x=96$

32. E. $M(n)=6$ if $n=16,23,32,61$, additionally $M*(n)=6$ if $M(n)=16,32$ so $n=28,44,48,82,84$, or if $M(n)=28,48$ so $n=47,68,74,86$.

33. D. $\frac{1}{4}+\frac{1}{16}+\frac{1}{36}+\frac{1}{64}+\cdot=\frac{1}{4}+\frac{1}{4}(\frac{1}{4}+\frac{1}{9}+\frac{1}{16}+\frac{1}{25}+\cdots=\frac{1}{4}+\frac{1}{4}s=\frac{s+1}{4}$

34. A. $x^2-2x+2=2x^2-6x-10$, so $x^2-4x-12=(x-6)(x+2)=0$, so the x values are $-2,6$ and the slope is $\frac{6^2-2(6)+2-[(^-2)^2-2(^-2)+2]}{6-^-2}=\frac{16}{8}=2$

35. B.

4	2	3	1
3	1	2	4
2	4	1	3
1	3	4	2

36. C. The points of intersection are $(5,0),(-3,4)$, the distance is then $4\sqrt{5}$

37. D. $x^2-2ax-6a=x^2-2a(x+3)=b(x+3)^2+3^2=bx^2+6bx+9b^2+9$, $b=1, a=-3$, so $b-2a=7$

38. C. $a+b+c+d=15, a-b+c-d=3, a+b-c-d=7$, $1.5(a+b+c+d)-.5(a-b+c-d)-1(a+b-c-d)=b+2c+3d=1.5(15)-.5(3)-7(1)=14$

39. E. $x+\frac{800}{x}+\sqrt{x^2+\frac{640000}{x^2}}=100,$

$100x=x^2+800+\sqrt{x^4+640000}$, $(x^2-100x+800)^2=x^4+640000$

$x^4-200x^3+11600x^2-160000x+640000=x^4+640000,$

$0=200x^3-11600x^2+160000x$, $0=x^2-58x+800,$

$x=29+\sqrt{58^2-4(800)}=29+\sqrt{41}$

$\sqrt{x^2+\frac{640000}{x^2}}=\sqrt{841+41+58\sqrt{41}+(882-58\sqrt{41})}=\sqrt{1764}=42$

40. D. Set 15 aside that produce the prey offspring, the remaining 5 have their population double each year, so there are $15+5\cdot 2^t=5(3+2^t)$.

Note: problems 28 and 39 differ from the key provided at the bottom of the original file, the rest agree.