

State Mathematics Finals: Geometry

May 6, 2004

- Given two similar triangles one of which has twice the perimeter of the other, by what factor is the area of the larger triangle bigger than the smaller?
a. 2 b. 4 c. $\sqrt{2}$ d. $2\sqrt{2}$ e. none of these
- Find the size of the smaller angle of an isosceles triangle which has the following properties: one angle is half the size of another, and none of the angles are 90° .
a. 36° b. 45° c. 60° d. 72° e. none of these
- To get a cable from the computer to the internet hub, Adam lays the cable carefully along the edge of the carpet by the wall. Assuming the room is rectangular, if he needed 3 feet of cable to get from the hub to the floor, 12 feet of cable to run along the north wall, 14 feet along the east wall, 7 feet along the south wall, and 5 feet up to the desk top computer, approximately what is the straight line distance from the hub to the computer?

a. 15 ft b. $\sqrt{393}$ ft c. 21 ft d. 41 ft e. none of these

- Three circles each with radius r intersect each other such that each passes through the center of the other two circles. Find the area of the intersection of their interiors.

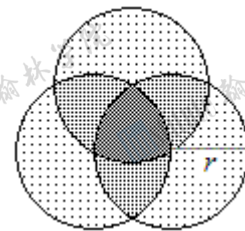
a. $\frac{\pi r^2}{6}$

b. $\left(\frac{\pi}{3} - \frac{\sqrt{3}}{4}\right)r^2$

c. $\frac{\sqrt{3}\pi r^2}{3}$

d. $\left(\frac{\pi - \sqrt{3}}{2}\right)r^2$

e. $\left(\frac{\pi + \sqrt{3}}{6}\right)r^2$



- The last cookie is gone, and either Chris, Pat, Lou, Sam or Sam's dog ate it. The following is known: Each one (except Sam's dog) said they knew who ate the cookie; Chris always tells the truth; Pat never tells the truth; Sam said it was either Pat, Chris or Lou; Lou said, "It wasn't me"; Pat claimed Sam was a liar; and Chris responded that Pat was the only liar. Who stole the cookie?

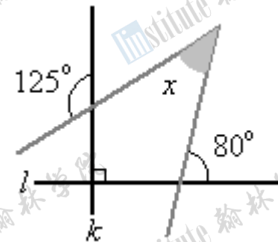
a. Chris b. Pat c. Lou d. Sam e. The dog

6. If the diameter of a cylindrical can is increased by 20%, by approximately what percentage should the height be increased in order to double the volume of the can?

a. 80.0% b. 66.7% c. 50.0% d. 41.4% e. 38.9%

7. Assume that $l \perp k$ and the angles are as shown. Determine $m\angle x$.

a. 20° b. 25° c. 45°
d. 65° e. none of these



8. The points with coordinates $(0,1)$, $(3,5)$ and $(5,0)$ are the vertices of a triangle. What is the area of that triangle?

a. 25 sq. units b. 16.5 sq. units c. 15 sq. units d. 12.5 sq. units e. none of these

9. A large window composed of a rectangle and semicircle, as shown, has an area of $16,000 \text{ cm}^2$. If the window has a width of 80 cm, how high is it?

a. 200 cm b. $200 - 10\pi$ cm c. $240 - 10\pi$ cm

d. $\sqrt{120} + \frac{40}{\pi}$ e. $\sqrt{240} - \frac{40}{\pi}$

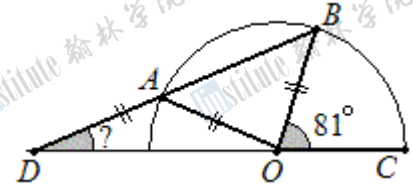


10. Find an expression for the perimeter of a rectangle in terms of its area A and the length of its diagonal, d .

a. $\frac{4A}{d}$ b. $2(\sqrt{2A} + d)$ c. $\sqrt{4d^2 + 2A}$ d. $2(\sqrt{d^2 + 2A})$ e. none of these

11. In the figure on the right $DA=AO=BO$ and $m\angle BOC = 81^\circ$. What is $m\angle ADO$?

a. 19° b. 27° c. 20.5°
d. 54° e. none of these



12. Starting at noon, how long will it take for the hour and minute hands of a typical analog clock to form a 90° angle?

a. 15 minutes b. $16\frac{1}{3}$ minutes c. $16\frac{23}{60}$ minutes d. $16\frac{5}{12}$ minutes e. $16\frac{4}{11}$ minutes

13. Given a circle centered at (3,4) that passes through point (7, 1), which of the following is the equation of the tangent line to the circle at point (7,1)?

a. $4x - 3y = 25$ b. $3x + 4y = 25$ c. $3x - 4y = 17$
d. $4x + 3y = 31$ e. none of these

14. In a hypothetical island the following is true:

1. Any two towns are directly connected by a single highway.
2. Intersections of highways occur only at towns.
3. Every town has exactly three highways.
4. Every highway goes to (or through) exactly three towns.

How many towns are on the island?

a. 4 b. 6 c. 7 d. 9 e. can't be determined

15. Consider a pyramid of unknown height and a 12×12 meter square base. If the height is increased by 2 meters, the lateral surface area is increased by 24 square meters. How high is the original pyramid? (The lateral surface does not include the pyramid's base.)

a. 2.5 meters b. 4.5 meters c. $\frac{3\sqrt{5}}{4}$ meters d. 5.625 meters e. none of these

16. Modify the right triangle with sides 11, 60, and 61 by doubling the length of its shortest side, but keeping its perimeter and largest angle unchanged. What is the length of the new triangle's hypotenuse?

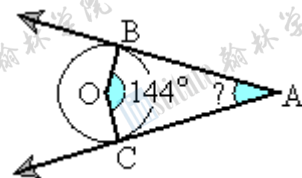
a. 52.8 b. 12.5 c. 55 d. $\sqrt{2885}$ e. 57.2

17. Given two intersecting circles $(x-2)^2 + (y-1)^2 = 4$ and $(x-3)^2 + (y-4)^2 = 9$, which of the lines listed below pass through the points of intersection?

a. $3x - y = 5$ b. $2x + 6y = 15$ c. $10x + 5y = 13$
d. $x + 3y = 15$ e. none of these

18. Rays \overrightarrow{AB} and \overrightarrow{AC} are tangent to a circle centered at point O . If $m\angle BOC = 144^\circ$, find $m\angle BAC$.

a. 36° b. 44° c. 18°
d. 24° e. Cannot be determined



19. Given three sets A , B , and C for which the following is true. \overline{A} indicates the complement of A .

i. $(A \cap B) \cup C \equiv \{1, 2, 3, 4, 5\}$

ii. $A \cup (\overline{B} \cap C) \equiv \{2, 3, 6, 7, 8\}$

iii. $B \subset C$

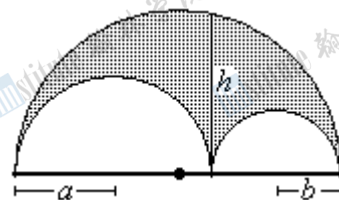
If the total of the values in set A is twice the total of those in set B , what are the elements of B ?

a. $\{2, 3, 5\}$ b. $\{1, 4, 5\}$ c. $\{1, 2, 4, 5\}$ d. $\{1, 3, 4, 5\}$ e. $\{1, 2, 3, 4, 5\}$

20. A wheel with a 70 centimeter diameter is rolled down a hill at a constant speed of 30 kilometers per hour. If it rolled for 4 minutes how many complete revolutions did it make?

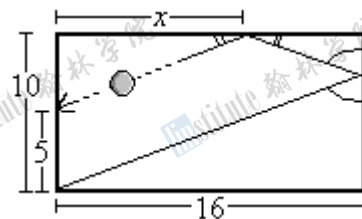
a. 545 b. 1,099 c. 454 d. 1,428 e. 909

21. An *arbelos*, pictured on the right, is the region formed by three mutually tangent circles whose centers are collinear. If the radii of the two smaller circles are a and b , and h is the height of the larger circle at the tangent point of the two smaller ones, what is the area of the *arbelos*?



- a. $\frac{\pi h^2}{4}$ b. $\frac{\pi h(a+b)}{2}$ c. $\frac{\pi h \sqrt{a^2 + b^2}}{2}$ d. $2\pi ab$ e. $\frac{\pi \sqrt{h^2 + (a+b)^2}}{4}$
22. In order to look at the peak of a certain mountain 30 km in the distance one needs to angle the telescope at 3° over the horizontal. If the elevation of the observer is at 633 meters above sea level, how high is the mountain?
- a. 790 m b. 1,572 m c. 2,205 m d. 2,790m e. none of these

23. Given a $16 \text{ ft} \times 10 \text{ ft}$ room in which a ball is kicked against a wall, and eventually hits the midpoint of the opposite wall. Assuming that the angles from bounces are preserved and that the ball's path is as shown, find x .



- a. $7\frac{1}{2}$ ft b. 8 ft c. 10 ft d. $10\frac{1}{2}$ ft e. none of these
24. A *Pythagorean Triple* is a set of three positive integers that satisfy the equation $a^2 + b^2 = c^2$. If the smallest of the three is 21 and the difference of the other two is 3, find the area of a triangle that can be constructed with these lengths.
- a. 1512 b. 787.5 c. 252 d. 756 e. none of these

25. A conical paper cup with height 16 cm and a 12 cm diameter for its top is filled to the top with water. If a fifth of the liquid is drunk by what percentage did the water level drop? (Round off error to the nearest 0.1%.)
- a. 6.7% b. 7.7% c. 15.4% d. 20% e. none of these

26. The smallest angle in a right triangle with sides, 28, 45, and 53 is approximately:

a. 27.8° b. 31.9° c. 38.5° d. 51.5° e. 58.1°

27. Consider a quadrilateral whose vertices A, B, C , and D , are on a circle. Let x, y , and z , be the truth values of the following three statements. What is the value of the ordered triple (x, y, z) ?

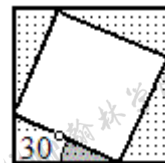
x : For quadrilateral $ABCD$ $m\angle ABC^\circ + m\angle CDA^\circ = 180^\circ$

y : The perimeter of quadrilateral $ABCD$ is greater than twice the diameter of the circle.

z : The perpendicular bisector of any side will pass through the circle's center.

- a. (F, F, T) b. (F, T, T) c. (T, T, T) d. (F, F, F) e. (T, F, T)

28. The picture to the right shows two square regions for which the vertices of one square lie on the sides of the other. If the angle of tilt of the inner square is 30° , what part of the outer square is covered by the inner one?



- a. 0.5 b. $4 - 2\sqrt{3}$ c. $\frac{\sqrt{3}-1}{2}$ d. $\frac{\sqrt{3}+1}{5}$ e. $2\sqrt{3}-3$

29. Given the following two sequences, find the smallest value of m/n such that $a_m = b_n$.

$$a_1 = 25, \quad a_2 = 50, \quad a_3 = 75 \dots$$

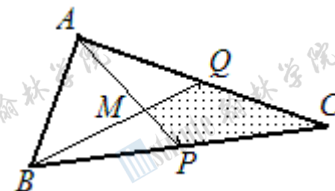
$$b_1 = 1, \quad b_2 = 3, \quad b_3 = 6, \quad b_4 = 10, \dots$$

- a. 0.5 b. 0.52 c. 1.0 d. 1.04 e. $0.458\bar{3}$

30. What is the measure to the nearest tenth of a degree of the largest angle of a triangle with sides of length 2, 2, and $1 + \sqrt{5}$?

- a. 72° b. 72.8° c. 103.7° d. 107.2° e. 108°

31. $\triangle ABC$ is divided into four parts by the segments connecting vertex A with the midpoint of \overline{BC} and vertex B with the midpoint of \overline{AC} . If the area of $\triangle ABC$ is 126, what is the area of quadrilateral $CPMQ$?



- a. 63 b. $21\sqrt{6}$ c. 21
d. 42 e. can't be determined

32. Given a triangle whose angles have the following criteria: one angle is twice as large as the other, and the third is 45° smaller than the sum of the other two. How large is the largest of the three angles?

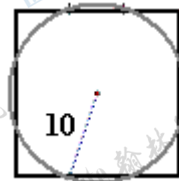
- a. 67.5° b. 75° c. 90° d. 135° e. none of these

33. A rectangle with integer lengths has a 1 unit wide border. Find the area of the smallest rectangle for which the region covered by the outer border is the same as the region covered by the actual rectangle.

- a. 30 b. 28 c. 24 d. 21 e. 18

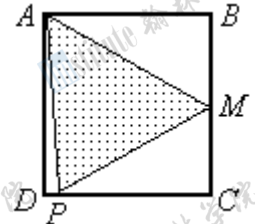
34. Find the area of a square whose four sides are trisected by a circle with radius 10.

- a. 360 b. 314 c. 400
d. $280 + 20\sqrt{3}$ e. none of these



35. Find the ratio of $\overline{AB} : \overline{BC}$ for the rectangle $ABCD$, for which A , M (the midpoint of \overline{BC}), and P (a point on \overline{CD}) form an equilateral triangle.

- a. $1 : 1$ b. $\sqrt{3} : 2$ c. $\sqrt{3} : (\sqrt{5} - 1)$
d. $(\sqrt{5} + 1) : (\sqrt{5} - 1)$ e. $(\sqrt{3} + 1) : 2$



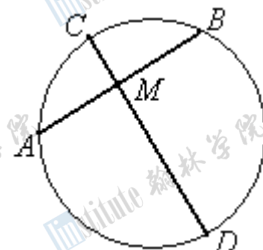
36. A survey of students yielded the following results. Of the female students surveyed, 25% had traveled overseas at some point in their lives, while 192 had not. Of the male students 184 said they have never traveled overseas. If an equal number of male and female students travel abroad, how many people were in the survey?

- a. 376 b. 470 c. 472 d. 504 e. 512

37. If a hot water heater is 20 meters from the sink, and the tubing has a 1 cm diameter, approximately how long will it take for the hot water to reach the sink if the flow rate is 2.8 liters a minute? (A liter is $1,000 \text{ cm}^3$.)

a. 43 sec. b. 34 sec. c. 107 sec. d. 11 sec. e. none of these

38. Let \overline{AB} be a chord of length 16 that is bisected by a second chord, \overline{CD} . How long is \overline{CD} if \overline{DM} is 3 times as long as \overline{CM} ?



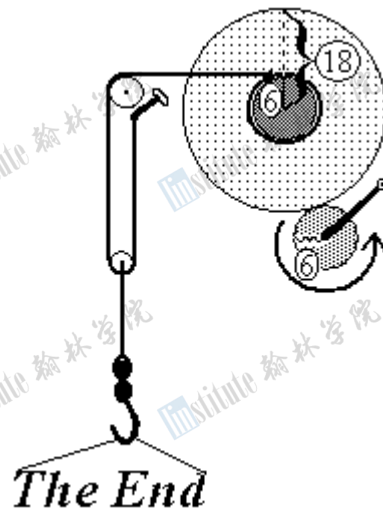
a. $\frac{8\sqrt{3}}{3}$ b. $\frac{64}{3}$ c. $8\sqrt{3}$
d. $\frac{32\sqrt{3}}{3}$ e. $\frac{20\sqrt{2}}{3}$

39. The following three points define the vertices of a triangle: $(0, 0)$, $(4, 10)$ and $(10, 6)$. Find the expression of the horizontal line that divides this triangle into two pieces of equal area.

a. $y = \sqrt{30}$ b. $y = 5$ c. $y = 6$ d. $y = 5.5$ e. $y = 3 + \sqrt{19}$

40. Two wheels drive a pulley as shown in the figure on the right. The wheel with the crank has a radius of 6 inches. It drives a wheel of radius 18 inches and at the larger wheel's center is a spool with a 6 inch radius. Approximately how many inches will the hook raise the sign if you turn the crank 5 times counter clockwise?

a. 5π in b. 10π in c. 15π in
d. 20π in e. 30π in



The End