multilite m # 3 Institute ## mistime ## institute \$ institute # Part I: Multiple Choice (20 Problems) Y. institute # 1. When $\frac{5x^2 + 20x + 6}{x^3 + 2x^2 + x}$ is decomposed into partial fractions, with each term reduced to lowest terms, the sum of the numerators is d. 12 . -4 a. 16 c. 14 2 A person deposits \$500 into a savings account at the end of every month for 4 years at 6% annual rate compounded monthly. How much interest will be earned during the 4 years? Y. d. \$3048.92 e. \$4098.46 a. \$1440 c. \$2024.39 b. \$1480.27 3. There are 100 members of the senate, 2 from each state. In how many ways can a committee of 5 senators be formed if no state may be represented more than once? c. 4,950 d. 67,800,320 e. 254,251,200 N/s a. 2,118,760 b. 75,287,520 4. You have 6 sticks of lengths 10, 20, 30, 40, 50, and 60 centimeters. The number of non-congruent triangles that can be formed by choosing three of the sticks to make the sides is a. 3 b. 6 d. 10 e. 12 c. 7 5. A glass box 7 cm \times 12 cm \times 18 cm, closed on all six sides is partly filled with colored water. When the box is placed on one of its 7×12 sides the water level is 15 cm above the table. When the box is placed on one of its 7×18 sides the water level above the table, in centimeters, will be c. 10 withit the a. 7.5 d. 12.5 b. 9 e. none of these 6. Two integers are said to be partners if both are divisible by the same set of prime numbers. The number of positive integers less than 25 that have no partners less than 25 is withthe # a. 11 c. 13 d. 16 💯 b. 12 e. 24 7. There are four cottages on a straight road. The distance between Ted's and Alice's cottages is 3 kilometers. Both Bob's and Carol's cottages are twice as far from Alice's as they are from Ted's. In kilometers, the distance between Bob's and Carol's cottages is withit mark 's PC Y. ittill b. 2 3 itute # a. 1 2004 North Carolina State Mathematics Contest Page 1 of 6

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mutilite # # " Institute # # " Institute \$ 75 'S multille m 26 - 3 mating m # 3 Institute \$7 \$7 'S 而如此他称林等除 15. The sum of the two largest numbers x for which the determinant $\begin{vmatrix}
2x-2 & 1 & 4 \\
6x-11 & 2x-5 & 2x+5 \\
-2x+2 & -1 & x-2
\end{vmatrix}$ equals zero is matitute # # '3 PR Y. e. none of the above a. 20 b. 5 c. 2 d. $\frac{-1}{2}$ e. none of the above 16. Consider the circles with radii $4\sqrt{5}$ and which are tangent to the line x - 2y = 20 at matine # # the point (6, -7). The sum of the x coordinates of the centers of the circles is a. 12 b. -14 c. 3 d. -5 e. 2 17. Given the equation $x^3 - 2x^2 + x - 3 = 0$, an equation whose roots are each 2 less than Y. the roots of the given equation is $-8x^{2} + 21x - 21 = 0$ b. $x^{3} - 4x^{2} - x - 5 = 0$ c. $x^{3} - 4x^{2} + 2x - 6 = 0$ d. $x^{3} + 4x^{2} + 5x - 1 = 0$ e. $x^{3} + 4x^{2} - 2x + 6 = 0$ a. $x^3 - 8x^2 + 21x - 21 = 0$ Ro 18. An experiment consists of choosing with replacement an integer at random among the numbers from 1 to 9 inclusive. If we let M denote a number that is an integral multiple of 3 and N denote a number that is not an integral multiple of 3, which of the followa. M N N M N b. N M M N c. N M M N M d. N N M N e. M N M M Y. 19. An 8 foot by 8 foot area has been tiled by one foot square tiles. Two of the tiles were defective. What is the probability that the two defective tiles share an edge? astitute the the "3 PR N. b. $\frac{1}{12}$ c. $\frac{1}{16}$ d. $\frac{1}{18}$ e. $\frac{1}{64}$ 标准.1 20. In the diagram if QR = d, then PS equals a. $\frac{\sin(\beta)}{\sin(\alpha - \beta)}d$ b. $\frac{\tan(\beta)}{\tan(\alpha) - \tan(\beta)}d$ c. $\frac{d}{\tan(\alpha) - \tan(\beta)}$ d. $\frac{d}{\cot(\beta) - \cot(\alpha)}d$ No. Multule # # 'S e. $\frac{\sin(\alpha)\sin(\beta)}{\cos(\alpha)(\sin(\alpha) - \sin(\beta))}d$ N. α R

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TUNITING WAY 28 multille m # 3 multille m # " multine m ** * matinte m # " matinte m # 3 -2) · 注 院 matine # # B PR 而此此此称林谱院 Part II: Integer Answers (15 Problems) Y. multine # tute to ? 1. Find *n* so that matine # # 13 1% $\frac{1}{1+\sqrt{3}} + \frac{1}{\sqrt{3}+\sqrt{5}} + \frac{1}{\sqrt{5}+\sqrt{7}} + \dots + \frac{1}{\sqrt{2n-1}+\sqrt{2n+1}} = 100$ 2. If **tan 3***x* is written in terms of **tan***x*, $\tan 3x = \frac{A \tan x - B \tan^3 x}{1 - C \tan^2 x}$ withthe ## # 13 PR find A + B + C. multitute # # # Y. stitute the the 'S 3. Consider the equation 15x + 14y = 7. Find the largest four digit integer x for which there is an integer y so that the pair (x, y) is a solution. The Marth & R mittille # # ** Ro 4. Let P be the set of primes that divide 200! (i.e. 200 factorial). What is the largest integer k, so that the set of primes that divides k! is equal to P? 5. What is the remainder when $7^{348} + 25^{605}$ is divided by 8? malitute # ** Y. 6. How many possible values can there be for three coins selected from among pennies, nickels, dimes and quarters? 7. A water tank has been sanitized by pouring in chlorine bleach. Bleach is toxic at the N. level needed to sanitize, so you need to flush out the tank using clean water. The result is that after each hour of flushing there is a 19% reduction in the bleach concentration. As-sume that when you began fluching the bleach sume that when you began flushing, the bleach concentration is 150 mg/gal. You can safely use the water tank for drinking purposes when the bleach concentration is below 0.7 mg/gal. What is the minimum number of whole hours you should flush the tank for safe drinking purposes? niinte # ** ** 施法资税 Y. 2004 North Carolina State Mathematics Contest Page 5 of 6 to the the B to the the B. Ph to the the 1/3 Ph Y. the star sit 's

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