Part I: Multiple Choice (20 Problems)

1) Mr. Taylor, the terrible T, gave his class a test the day after spring break. The average score for all students was 80. The average score for those who passed was 84 and for those who failed was 60. If there were 60 students in the class, how many failed?

a) 12 b) 15 c) 13 d) 14 e) 10

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2) Which of the following triples could not be the sides of a triangle?

a) 7, 7, 8 b) 3, 4, 6 c) 2, 7, 11 d) 17, 18, 25 e) 1023, 2168, 3040

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3) If the graphs of $2y + x + 3 = 0$ and $3y + a\,x + 2 = 0$ are to meet at right angles, then a is:

a) -6 b) 6 c) $-2/3$ d) $3/2$ e) none of these

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4) In a circle of radius r, a central angle with measurement $4\frac{1}{2}$ radians subtends an arc of 108 ft. Find r.

a) 216 ft. b) 53 ft. c) 48 ft. d) 24 ft. e) 12 ft.

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5) Let ABC be an equilateral triangle with sides x. Let P be the point of intersection of the three angle bisectors. Find $\overline{AP}$.

a) $\frac{x\sqrt{3}}{3}$ b) $\frac{x\sqrt{3}}{6}$ c) $\frac{x\sqrt{3}}{3}$ d) $\frac{5x\sqrt{3}}{6}$ e) $\frac{2x\sqrt{3}}{3}$

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6) Evaluate the product $\log_3 16 \cdot \log_2 27$.

a) 8.75 b) 1.62 c) 12 d) 10.64 e) none of these

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7) There are two values of x, where $0 < x < B/2$, for which

$\tan\left(\frac{p}{4} + x\right) = 9\tan\left(\frac{p}{4} - x\right)$.

Find these two values of x in radians and calculate their product.

a) 1.107 b) 0.5133 c) 0.7512 d) 1.525 e) 0.9871

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8) Two ladders, one of which is twice as long as the other, rest on the floor and reach the same vertical height on the wall. The shorter ladder makes an angle of 60° with the floor. What angle does the longer ladder make with the floor? Round to the nearest degree.

a) 30° b) 15° c) 25° d) 26° e) 19°

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9) If $f(2x+1) = 4x^2 + 14x$, find the sum of the roots of $f(x) = 0$.

a) $9/4$ b) 5 c) $-9/4$ d) -5 e) none of these

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10) Solve $x^2 + x \cot A \cos A - 1 = 0$ for x given $0 < A < p/2$. Find the sum of the roots.

a) $\sin A - \tan A$ b) $\sin A - \csc A$ c) 1
d) $\cos A + \sec A$ e) none of these

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11) Find the sum of the real solutions of the equation $e^2 \cdot x^{\ln x} = x^3$ correct to 3 decimal places.

a) 14.778 b) 12.156 c) 10.107 d) 9.126 e) none of these

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12) Consider the equation $x^2 + kx + 1 = 0$. A single fair die is rolled to determine the the value of the middle coefficient, k. The value for k is the number of dots on the upper face of the die. The probability that the equation will have real, unequal roots is:

a) $1/3$ b) $2/3$ c) $1/2$ d) $3/4$ e) none of these

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13) Suppose $a = 212$ and $b = 2201$ are numbers expressed in the base 3 number system. The product ab expressed in the base 3 number system is:

a) 1102212_{three} b) 2110001_{three} c) 2002211_{three}
d) 2022012_{three} e) none of these

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14) Let $a * b$ be a binary operation defined on the integers by $a * b = b + 1$. Under what conditions on integer a, b, and c is $*$ associative?

a) $a = b = c$ b) a or b or c is zero c) a or b or c is -1
d) $a * b = -1$ e) The operation $*$ is never associative.

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15) Students in a class are selected at random, one after the other, from a class consisting of 3 boys and 4 girls. The probability that boys and girls in the class alternate starting with a girl first is:

a) 1/35 b) 34/35 c) 5/7 d) 6/7 e) 32/35

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16) An inlet pipe can (by itself) fill an empty tank in 2 hours and an outlet pipe (by itself) can drain the same tank when full in 5 hours. If the tank is half full when both valves for both pipes are opened, how long will it take to fill the tank?

a) 1 hr. 50 min. b) 1 hr. 40 min. c) 1 hr. 28 min.
d) 1 hr. 18 min. e) none of these

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17) Assume that p is a real number. In order for $\sqrt[3]{x+3p+1}-\sqrt[3]{x}=1$ to have real solutions, it is necessary that:

a) $p \ge \frac{1}{4}$ b) $p \ge -\frac{1}{4}$ c) $p \ge 0$ d) $p \ge \frac{1}{3}$ e) $p \ge -\frac{1}{3}$

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18) Find the smallest positive integer x so that $\tan\left(\tan^{-1}\frac{x}{10}+\tan^{-1}\frac{1}{x+1}\right)=\tan\frac{p}{4}$.

a) 8 b) 9 c) 7 d) 3 e) 0

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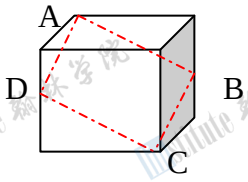
19) A pair of opposite vertices and the midpoints of a pair of opposite edges of a cube are connected to form a quadrilateral. If each edge of the cube has length k, find the area of the quadrilateral.



a) $\frac{\sqrt{3}\,k^2}{2}$ b) $\frac{\sqrt{6}\,k^2}{2}$ c) $\frac{\sqrt{2}\,k^2}{3}$ d) $\frac{k^2}{2}$ e) $\frac{\sqrt{6}\,k^2}{3}$

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20) The product of three consecutive positive integers is always divisible by 3. What is the probability that the product of three consecutive positive integers is divisible by 12?

a) $\frac{1}{6}$ b) $\frac{2}{3}$ c) $\frac{5}{6}$ d) $\frac{1}{2}$ e) $\frac{3}{4}$

Part II: Integer Answer (15 Problems)

1) How many subsets of $S = \{1, 2, 3, \cdots, 10\}$ contain a 1, 2 or 3?

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2) Three parallel lines ℓ_1, ℓ_2, and ℓ_3 are drawn through the vertices A, B, and C of a square ABCD. If the distance between ℓ_1, and ℓ_2 is 7 and between ℓ_2, and ℓ_3 is 12, find the area of ABCD.

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3) A weather plane flies due north into a 40 mph wind until it reaches the North Pole, at which point it turns around and returns to its original position. The pilot notes that his average velocity for the round trip was 396 mph. even though the plane was flying at its maximum velocity the entire time. What is the maximum velocity of the plane when there is no wind? Answer in mph.

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4) Find the sum of a and b if $\frac{2}{9!}+\frac{2}{7!\cdot 3!}+\frac{1}{5!\cdot 5!}$ is written in the $\frac{2^a}{b!}$ form where $a+b$ is as small as possible.

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5) Evaluate: $A = \frac{100}{\log_2 100!}+\frac{100}{\log_3 100!}+\cdots+\frac{100}{\log_{100} 100!}$.

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6) Note that $\frac{1}{7} = 0.142857\overline{142857}$ is a repeating decimal with 6 distinct digits in the period. Find the sum of the six smallest positive integers n such that $\frac{1}{n} = .abcdef\overline{abcdef}$ and a, b, c, d, e and f are all different.

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7) Given that a and b are integers, find $b-a$ so that $1+\sqrt{2}$ will be a root of $x^4+ax+b=0$.

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8) In ΔABC $(a+b)(a-b) = c(b+c)$, find the measure of angle A to the nearest degree. Assume that side a is opposite $\angle A$.

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9) The 1st, 2nd and 3rd perfect squares are underlined in the following arithmetic sequence, 1, 8, 15, 22, 29, 36, 43, 50, 57, 64, ⋯. Find the 100th perfect square in the sequence.

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10) If $k \neq 0$ and the roots of $kx^3 - 4x^2 + 6x - 1 = 0$ are a, b, and c, find the value of $\frac{1}{ab} + \frac{1}{bc} + \frac{1}{ac}$.

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11) Find k given that $\begin{cases} f(0) = k \\ f(n) = f(n+1) - 3n - 2 \end{cases}$ and $f(-50) = 4000$.

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12) Find the coefficient of x^3 in the expansion of $(1 + x + x^2)^{12}$.

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13) How many times does the curve with equation $y = \cos x$ intersect the curve with equation $y = \frac{100 - x}{100}$?

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14) The tips of a five-pointed star are to be painted red, white and blue. How many ways can this be done if no adjacent points can be the same color?

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15) Find the last 3 digits of 7^{1166}.
