multilite m # " Institute # # " multilite # \*\* \* Institute \$ 75 'S Institute \$5 \$7 'S Institute # # 3 Page 1 of 5 2002 North Carolina State Mathematics Contest 训册称林谱院 Let f(x) be a quadratic polynomial such that f(3) = 15 and f(-3) = 9. Find the coefficient of x in f(x). Y. Part I: Multiple Choice (20 Problems) 1. a) 2 b) 3 c) -2 d) -3 e) 1 Find the sum of all values of x so that  $16^{(x^2+3x-1)} = 8^{(x^2+3x+2)}$ . a) 2 Astitute the the c) -3 d) -5 b) 3 a) 0 e) none of these Find the sum of all values of x > 0 for which  $(\log_{27} x^3)^2 = \log_{27} x^6$ . a) 1 stitute b) 5 🚮 c) 0 e) none of these d) 10 If |x| - x + y = 10 and x + |y| + y = 12, then x + y is equal to 4. mutute # # 3 PS multille ## # 'S PS a)  $\frac{26}{5}$  b)  $-\frac{26}{5}$  c) 0 d)  $\frac{42}{5}$  e) none of these  $x^{3}$  and  $x^{5}$  and  $x^{5}$  and  $x^{5}$  by  $x^{4}$  and  $x^{2}$  and  $x^{6}$  by  $x^{6}$  $x^{6$ If  $x + \frac{1}{x} = k$ , find  $x^5 + \frac{1}{x^5}$ . Write the answer in the form of e) none of these stimute to the second secon R An equilateral triangle, with sides of 10 inches, is inscribed in a square ABCD in such a 6. way that one vertex is at A, another vertex on  $\overline{BC}$  and one on  $\overline{CD}$ . Find the area of the 面站抽版新林塔梯 ...ay thi square. b)  $25(2+\sqrt{3})$ c) 25 a)  $25(2-\sqrt{3})$  $\frac{100}{2+\sqrt{2}}$ d) e) none of these How many digits are in 17<sup>10,000</sup> ? a) 12,304 b) 1,700 c) 10,000 d) 12,305 e) none of these stitute the the 7. If  $\tan A + \tan B + \tan C = k$ , 0 < k < 10, and  $m \angle A + m \angle B + m \angle C = 180^{\circ}$ , find 8. tan A tan B tan C. a)  $\frac{1}{4}$  k b) 4k c)  $k^3$  d) k e) none of these to the the B. Ph to the the 'B We Y. · 标林 · 洛 to the the B to the the B 大学学

加根额样等除 Y. Page 2 of 5 2002 North Carolina State Mathematics Contest 9. Let S be a set with 100 elements. How many subsets does S have which contain at least 50 elements: c)  $1.167 \times 10^{30}$ b) 6.843 x 10<sup>29</sup>  $1.217 \times 10^{30}$ to all the state the a) institute 30 e) none of these d)  $1.009 \times 10^{29}$ It usually takes Johnny 8 hours to mow the yard. Frankie can do it in 6 hours. Johnny 10. got Frankie to help one Saturday morning, but when one-half of the lawn was mowed, 加加斯林道際 Frankie quit and Johnny had to finish by himself. How long did Johnny mow altogether Y. tingitute the th including the time he worked with Frankie? c)  $4\frac{4}{5}$  hrs. d)  $5\frac{5}{7}$  hrs. e) none of these b)  $5\frac{1}{3}$  hrs. a)  $5\frac{2}{5}$  hrs. Johnny figures that since the final exam counts as two tests, he only needs a 28 to have a 11. titute the the 's PL Y. 70 average. Even if he makes a 100 he will still only have an 88 average. What is the institute the th lowest score Johnny can make on the final and still have an 80 average? a) 65 d) 70 b) 66 c) 68 e) 72  $\begin{bmatrix} x \sec q + y \tan q = 2 \cos q \\ x \tan q + y \sec q = \cot q \end{bmatrix}$ , find what y equals. 山山市新林塔梯 12. If Institute ## #  $\cos 2q$ c)  $\cos q$ d) sin 2q e) none of these a) b)  $\sin q$ sin **q** Avitute the the 'S PR If the 14 blocks must be filled with whole numbers so that the sum of any three 13. tstitute ## consecutive blocks has a total of 18, find x. 8 a) 3 d) 4 b) 5 c) 7 e) 8 inte the the 's If  $(532)_b = 4(148)_b$ , for base b > 1, the b is in which range? 14. stitute the state 物" c) 11≤b≤15 b)  $6 \le b \le 10$ a)  $1 \le b \le 5$ d)  $16 \le b \le 20$ e) 21 ≤ b Given x \* y = x + y + 1, and  $a^{-1}$  means the inverse of a with respect to "\*". Calculate 前肌肉称林婆幣 15. Y. stitute the state  $(2*3)^{-1}*2^{-1}*3^{-1}$ b) -15 c) 15 a) -16 d) 16 e) none of these to the the B. Ph to the the By the R the the state 古 故 法 法 to the state

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7. Frankie and Johnny are sweethearts so Johnny always walks by to pick up Frankie on the way to school. If Johnny lives 3 blocks south and 4 blocks west of Frankie and Frankie lives 5 blocks west and 2 blocks south of the school, how many ways can Johnny walk to school if he will walk any route as long as he always walks north or east?

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- 8. The polynomial  $x^4 50x^2 + k = 0$  has four distinct real zeroes and these zeroes are equally spaced on the real number line. Determine the value of k.
- 9. Find the largest integer m so that  $5^7$  can be written as the sum of exactly m consecutive positive integers.

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- 10. Frankie and Johnny go skating at the track. Frankie likes to skate because she is much faster than Johnny is. Even when she takes the outside lane and gives Johnny the inside lane, she can make it around the track in 5 minutes, whereas it takes Johnny 6 minutes. So Johnny does not like to skate all that long. He notes that when they start they are lined up with the flagpole at the very center of the circular track and says that he will skate until the skaters and the flagpole lie on the same straight line. If they go in the same direction and maintain constant speeds, how many minutes do they skate?
  - 11. Find the sum of all positive integers n so that  $n^4 360n^2 + 400$  is a positive prime.
  - 2. Find the sum of the 4 positive integer values which make  $n^2 + n + 109$  a perfect square.

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- 13. The equation  $x^2 y^2 = 1,000,001$  has only two solutions where x and y are both in the set  $S = \{1, 2, 3, 4, ...\}$ . Find the smallest value of x in the solution set.
- 14. If one-hundred factorial (100!) base 10 is converted to base 6, how many zeroes will be at the end of the base 6 numeral?
  - 15. Find the smallest positive integer which has exactly 24 positive integer divisors.

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