Algebra I
State Mathematics Contest Finals, May 2, 2002

1. Solve for x, given $y = x^2 - 1$ and $x = 2y + 1$.

   a) $x \in \{0, 1\}$ b) $x \in \left\{-\frac{1}{2}, -\frac{3}{4}\right\}$ c) $x \in \left\{-\frac{1}{2}, 1\right\}$ d) $x \in \left\{0, -\frac{3}{4}\right\}$ e) none of these

2. George took three 100 point tests. The average of these three tests was 88. On the second test he missed half as many points as on the first test. His third test was 4 points higher than the second test. What was the grade on the third test?

   a) 88 b) 90 c) 98 d) 94 e) none of these

3. If $C = \frac{x - a}{x - b}$ then

   a) $x = \frac{bC - a}{C - 1}$ b) $x = \frac{C - a}{C - b}$ c) $x = \frac{C + a}{C + b}$ d) $x = \frac{1 - C}{a - bC}$ e) $x = \frac{C - b}{C - a}$

4. Given the first 10 values of a sequence { 1, 3, 4, 4, 5, 7, 8, 8, 9, 11, ... } find the sum of the sixteenth and seventeenth terms.

   a) 30 b) 33 c) 16 d) 35 e) none of these

5. Find the value for x so that the three points, {(2, 7), (6, 1), (x , 0)} are collinear.

   a) 7 b) $4\frac{1}{2}$ c) 10 d) $6\frac{2}{3}$ e) none of these

6. The cubic polynomial, $x^3 + 2x^2 - 19x + 12$, has three roots. Their product is:

   a) -2 b) -12 c) 12 d) $-3\sqrt{19}$ e) none of these

7. A right triangle with integer sides has area equal to 30. What is the length of its hypotenuse?

a) 10 b) 12 c) 13 d) 15 e) none of these

8. Let $f(x) = x^2 + x - 6$. For what values of t does $f(t-5) = 0$?

a) -3 and 2 b) -2 and 3 c) 5 d) 2 and 7 e) none of these

9. If x and y are non-negative, $y \le 6 - x$ and $y \ge x + 4$ then find the largest possible value of $2x + 3y$.

a) 42 b) 17 c) 24 d) 132.5 e) 18

10. The product of three consecutive positive integers is eight times their sum. Find the value of the middle number.

a) 0 b) 2 c) 3 d) 4 e) none of these

11. Starting at 12:00 noon how long will it take for the minute hand to make a right angle with the hour hand?

a) 15 minutes b) 18 minutes c) $16\frac{4}{11}$ minutes d) $16\frac{11}{60}$ minutes e) $16\frac{37}{60}$ minutes

12. Given: $3x - 4y = 7$ and $x + cy = 13$, for what value of "c" will the two equations not have a solution.

a) $\frac{3}{4}$ b) $\frac{4}{3}$ c) -4 d) $\frac{-4}{3}$ e) none of these

13. Solve for x. $2^{x+1} = 8^x$

a) 1 b) 3 c) $\frac{-1}{3}$ d) $\frac{1}{2}$ e) none of these

14. At noon Papa George was in the habit of placing 8 pennies into a jar. A short while later little Joe would take half of the pennies in the jar. If there were 9 pennies in the jar on Friday afternoon, how many pennies were there on the Monday morning of that week (i.e. four days earlier)?

a) 45 b) 24 c) 288 d) 248 e) 40

15. Given $p = \frac{m^2 - a\,t}{z}$ find the expression for t.

a) $t = \frac{p\,z - m^2}{a}$ b) $t = \frac{m^2 - pz}{a}$ c) $t = \frac{p\,z}{a} - m^2$ d) $t = a\,p\,z - \frac{m^2}{a}$ e) none of these

16. For $c>0$, solve for x: $\sqrt{2x + c} = x$.

a) $x = 1 + \sqrt{1 + c}$ b) $x = \sqrt{c}$ c) $x = 1 \pm \sqrt{c}$ d) $x = \sqrt{1 + c}$ e) $x = \frac{1 \pm \sqrt{c}}{2}$

17. Which of the following polynomials below is identical to the polynomial $(x^2 - 1)(x^2 - 4)$?

a) $x^4 + 4$ b) $x^2 - 5x + 4$ c) $(x^2 + 1)(x^2 + 4)$
d) $(x^2 + x - 2)(x^2 - x - 2)$ e) $x^4 - 4$

18. Find the distance between the two points defined by the intersection of $y = x^2 - x$ and $y = 3x + 5$.

a) $2\sqrt{130}$ b) 6 c) 18 d) $6\sqrt{10}$ e) $\sqrt{10}$

19. If $A^2 = B$, $A^3 = C$, $A^4 = D$, $A^5 = E$, and $A^6 = A$, then $B^2\,C^3$ is equal to:

a) A b) B c) C d) D e) E

20. The sum of two numbers is 42 and their product is 36. Which equation below will have the two numbers as a solution?

a) $x^2 - 42x + 36 = 0$ b) $x^2 - 42 = 36$ c) $x^2 - 36x + 42 = 0$
d) $x^2 - 6x + 78 = 0$ e) $(x - 42)(x - 36) = 0$

21. Given that A, B, C, and D are positive integers where $3A = 7B$, $5C = 4D$, and $2C = 11A$. Arrange the four integer variables in increasing order.

a) $ABCD$ b) $DCBA$ c) $BACD$ d) $DCAB$ e) $CADB$

22. Given that you have an unlimited number of nickels, dimes, and quarters, how many different combinations of these coins are there if the total is to be 65¢?

a) 11 or less b) 12 c) 13 d)14 e) 15 or more

23. The difference of two numbers is 2. If the difference of their squares is 18, what is their sum?

a) 20 b) a number between 10 and 16 c) a number between 5 and 8

d) this value can not be determined e) none of these

24. If $f(x) = 5 + 2x$ evaluate $f^{-1}(-3)$.

a) -4 b) -1 c) $-6\frac{1}{2}$ d) $-5\frac{1}{2}$ e) none of these

25. Given that 11 is the maximum value of the function $f(x) = -x^2 + 6x + c$ find the value for "c".

a) 0 b) 1 c) 11 d) $\frac{11}{6}$ e) 2

26. Troy's test average (based on 5 exams) is 83.2, and he wants to raise his grade to an 85. What is the minimum score he needs on the sixth test to achieve his goal?

a) 87 b) 90 c) 94 d) 98 e) none of these

27. Two fair dice are rolled. What is the probability that the difference of the squares of the numbers is divisible by three?

a) $\frac{1}{3}$ b) $\frac{11}{18}$ c) $\frac{1}{2}$ d) $\frac{5}{9}$ e) $\frac{2}{3}$

28. Given $\left(x^2-5x+5\right)^2-1=0$ Find the difference between the largest and smallest solution to this equation.

a) 3 b) 4 c) $2+\sqrt{5}+\sqrt{3}$ d) $\sqrt{5}$ e) none of these

29. Given $x^2=3^8$. Find x.

a) 9 b) 27 c) 81 d) $27\sqrt{3}$ e) none of these

30. Let $xy=13$ and $x-y=7$, find the value of x^2+y^2.

a) 33 b) 23 c) 120 d) 75 e) 49

31. Define the operation $\otimes$ as $x\otimes y=xy(x-y)$. Find x where $x\neq 0$ and $x\otimes 7=x$.

a) $1\frac{1}{7}$ b) $7\frac{1}{7}$ c) 7 d) $\frac{1}{7}$ e) equation has no solution

32. Define the operation $\otimes$ as in the previous problem, $x\otimes y=xy(x-y)$. Which of the following expressions is equivalent to $x\otimes(a+b)$.

a) $(x\otimes a)+(x\otimes b)$ b) $(x\otimes a)-(x\otimes b)$ c) $(x\otimes a)+(x\otimes b)-2abx$

d) $(x\otimes a)-a+(x\otimes b)-b$ e) $b(x\otimes a)+a(x\otimes b)$

33. Solve for x: $3=\dfrac{2^x+7}{3-2^x}$

a) $\frac{1}{2}$ b) $\log_2 3$ c) $\log_2\frac{3}{7}$ d) $\frac{-1}{2}$ e) -1

34. Which of the following equations has $\sqrt{c^2+1}$ as a solution for x ?

a) $c=x+\dfrac{1}{x-c}$ b) $x=c-\dfrac{1}{x+c}$ c) $c=\sqrt{x-1}$ d) $c=x-\dfrac{1}{x+c}$ e) $c=\dfrac{1}{\sqrt{x-1}}$

35. Find the equation of a line that divides the line segment connecting points $(1, 3)$ and $(7, -1)$ into halves, and is parallel to the segment that connects points $(1, 3)$ and $(-1, -1)$.

a) $y = 2x - 7$ b) $y = 2x + 1$ c) $y = -2x + 9$ d) $y = -2x + 5$ e) none of these

36. There are 4 female and 3 male students hoping to go on a trip. If two people are chosen at random from this group what is the probability that one female and one male student will be picked?

a) $\frac{2}{7}$ b) $\frac{3}{7}$ c) $\frac{1}{2}$ d) $\frac{4}{7}$ e) $\frac{1}{3}$

37. Given the parabola $y = x^2 - 4x + 6$ and the line $y = mx$, for what positive value of m will the line be tangent to the parabola?

a) -4 b) $-4 + \sqrt{20}$ c) $-4 + 2\sqrt{6}$ d) 1 e) $\sqrt{40}$

38. Find the non-zero solution for x: $8^x = \left(\frac{1}{4}\right)^{x^2}$

a) $\frac{2}{3}$ b) $\frac{-2}{3}$ c) 32 c) 2 e) $\frac{-3}{2}$

39. If $0 < \frac{x}{y} < 1$ which of the following must be true?

a) $x < y < 0$ b) $x < 1$ and $y > 1$ c) $0 < xy < y$

d) $0 < x < y$ e) $1 < \frac{y}{x}$

40. x is twice as large as y and y is 10 units bigger than z and z is 3 times as big as x, what is the value of $x^2 + y^2 + z^2$?

a) 164 b) 111 c) 324 d) $\frac{4100}{49}$ e) cannot be determined