Part I: Multiple Choice (20 Problems)

1. A driver travels from New York to Los Angeles and averages 40 mph. Since the driver has seen the sights she averages 60 mph on the way back from Los Angeles to New York. What was her average speed for the round trip?
a. 52 b. 53.5 c. 50 d. 48 e. none of these

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2. If the point $P=(u,v)$ is on the graph of $y=ax^2+bx+c$, $a\neq 0$, which of the following is also on the graph?
a. $\left(\frac{b}{a}-u, v\right)$ b. $\left(-\frac{b}{a}-u, v\right)$ c. $\left(-\frac{b}{a}+u, v\right)$
d. $\left(\frac{b}{a}+u, v\right)$ e. none of these

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3. Suppose the number $N = x5{,}399{,}84y$ is a positive integer with two unknown digits x and y and N is a multiple of 198. Find the units digit of $N \div 198$.
a. 5 b. 6 c. 7 d. 8 e. 9

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4. Solve the following pair of equations for x and y.
$x^2+xy+y^2=84$
$x-\sqrt{xy}+y=6$
What is the product of x and y?
a. 36 b. 25 c. 16 d. 64 e. 49

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5. Find the sum of the coefficients in the expansion of $(2a+b-c)^8$.
a. 720 b. 256 c. 676 d. 512 e. none of these

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6. Given $-\frac{\pi}{2}\leq \sin^{-1}x \leq \frac{\pi}{2}$, then $\tan(\sin^{-1}x)$ must equal to:
a. $\frac{x}{1-x^2}$ b. $\frac{x}{x^2-1}$ c. $\frac{x}{\sqrt{1-x^2}}$ d. $\frac{x}{\sqrt{x^2-1}}$ e. none of these

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7. Find the sum of the digits of the smallest prime factor of 1,111,111.
a. 15 b. 14 c. 13 d. 12 e. 11

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8. If all possible solutions to $\log_4(3-x)+\log_{0.25}(3+x)=\log_4(1-x)+\log_{0.25}(2x+1)$ are found, there will be
a. 2 positive solutions b. 2 negative solutions
c. only 1 positive solution d. 1 positive and 1 negative solution
e. none of these

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9. The area of triangle ABC is equal $a^2 + b^2 - c^2$. If angle C is acute, compute the numerical value of its secant where a, b, and c are positive real numbers.

a. $\frac{a}{bc}$ b. $\frac{ac}{b}$ c. $\frac{ab}{c}$ d. $\sqrt{17}$ e. impossible to determine

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10. How many right triangles, whose sides are all positive whole numbers, have the property that the area is numerically equal to its perimeter?

a. 2 b. 1 c. 0 d. 4 e. infinitely many

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11. Find the slope of a line with a positive rational slope, which passes through the point (6,0) and at a distance of 5 from (1,3). Write the slope in the form $\frac{a}{b}$, where a and b are relatively prime. What is the sum of a and b?

a. 24 b. 23 c. 22 d. 21 e. none of these

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12. Given that a and b are positive integers, find the smallest value of b so that $\frac{5}{31} < \frac{a}{b} < \frac{7}{43}$.

a. 37 b. 35 c. 32 d. 39 e. none of these

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13. A gambler has in his pocket a fair coin and a two-headed coin. He selects a coin at random and flips it twice. If he gets two heads, what is the probability that this was the fair coin?

a. $\frac{1}{2}$ b. $\frac{1}{3}$ c. $\frac{1}{4}$ d. $\frac{1}{5}$ e. none of these

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14. In our present calendar, leap years occur every 4 years except for centuries that are not divisible by 400. For example 2000 was a leap year and 1900 was not. If we continue on the present calendar, which of the following statements is true about the first day of the century for the next 10,000 years given that the first day of 2001 (21^{st} century) was a Monday. No new centuries will begin on:

a. Thursday b. Tuesday or Friday c. Saturday
d. Monday e. Wednesday, Friday or Sunday

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15. Find the third side of a triangle given that the other two sides are 100m and 200m respectively and the median to the third side is $10\sqrt{61}$.

a. 160 b. $16\sqrt{10}$ c. 180 d. $20\sqrt{2}$ e. none of these

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16. An antique dealer packs a spherical globe of radius 10 inches. It is packed in a box, which will just hold it. To further protect the globe during shipping the dealer plans to place a Styrofoam ball in each corner of the box. To the nearest tenth of an inch, find the radius of the largest Styrofoam ball that will fit.

a. 2.7 in   b. 2.8 in   c. 2.9 in   d. 3.0 in   e. none of these

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17. Solve the following system of symmetric equations.

$$x^2 + 3xy + y^2 + 2x + 2y = 8$$
$$2x^2 + 2y^2 + 3x + 3y = 14$$

The sum of the four x values in the solution set correct to three decimal places is:

a. 1.833 b. -1.833 c. 0 d. 2.831 e. -2.831

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18. The number 5857 has the property that the first two digits is one greater than the second two digits. Find a four-digit number with this property, which is a perfect square. The sum of the digits of this number is:

a. 17   b. 18   c. 19   d. 20   e. none of these

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19. In how many ways can seven basketball players of different heights line up in a single row so that no player is standing between two people taller than herself.

a. 32   b. 48   c. 64   d. 96   e. none of these

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20. Evaluate the product of $\tan 1^\circ \cdot \tan 3^\circ \cdot \tan 5^\circ \cdots \tan 179^\circ$.

a. 1 b. -1 c. 0 d. $+\infty$ e. none of these

Part II: Integer Answer (15 Problems)

1. If $S = 1^2 - 2^2 + 3^2 - 4^2 + \cdots + 199^2 - 200^2$, find the absolute value of S.

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2. Given a function F(x) such that $F(0) = 2$ and $F(x^2+1) = F(x)^2 + 1$ for all x. Find $F(5)$.

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3. Two tugboats leave from opposite sides of a river and meet 700 yards from the east side. The boats continue traveling until they reach opposite banks of the river, reverse directions, and continue until they meet 300 yards from the west bank. The boats travel at constant but unequal rates of speed and the time reversing is negligible. How wide is the river?

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4. If $\cos 5x = A\cos^5 x + B\cos^4 x + C\cos^3 x + D\cos^2 x + E\cos x + F$, find the sum of A, B, C, D, E, and F.

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5. If
$$\begin{aligned} x - y + z &= 1 \\ y - z + u &= 2 \\ z - u + v &= 3 \\ u - v + x &= 4 \\ v - x + y &= 5 \end{aligned}$$
, find $x + y + z + u + v$.

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6. A helicopter pilot is ordered to leave from base A and land at base B which is 27 miles from base A in a direction 50° east of north. The orders also say that the pilot should fly due north and then turn toward base B. If the pilot flies at 120 mph and lands at B with exactly one hour of flying time, how far north did he fly? Round answer to the nearest whole number.

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7. Suppose all the points on the curve $x^2 + y^2 - 10x = 0$ are reflected about the line $y = x + 3$. Find the locus of points. Write the locus of the reflected points in the form $x^2 + y^2 + Cx + Dy + F = 0$. Find the sum $C + D + F$.

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8. Find the sum of all positive integers n so that $2001 + n^2$ will be a perfect square.

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9. How many digits are in the smallest number with the property that if the first digit is moved to the right so as to become the last digit, the new number will be $\frac{3}{2}$ times the original?

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10. **Destination:** Tombstone
An English tourist in the wild and woolly West was informed at the hotel that there were four different ways he could travel to Tombstone.
1. He could ride the stagecoach all the way. This included one stopover of thirty minutes at a certain way-house along the road.
2. He could walk all the way. If he started walking at the same time the coach left the hotel, the coach would beat him to Tombstone by one mile.
3. He could walk to the way-house and then take the coach. If he and the coach left the hotel at the same time, he would arrive at the way-house just in time to catch the coach.
4. He could take the coach to the way-house, then walk the rest of the way. This was the fastest procedure, getting him to Tombstone fifteen minutes ahead of the coach. How fast does he walk? Answer should be in miles per hour.

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11. In base ten the following three rules for divisibility hold.
1. A number which ends with an even number is exactly divisible by 2.
2. A number which ends in 0 or 5 is exactly divisible by 5.
3. If the sum of the digits is divisible by 3, then the number is exactly divisible by 3.

What is the next base for which all 3 of these rules work?

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12. Solve the equation $x^2 + y^2 + z^2 = 12(x + y + z)$ where x, y, and z are integers with $x \geq y \geq z$. How many solutions does the equation have?

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13. Let a, b, and c denote any three integers. Define $a * b$ in such a way that
1. $1 * 1 = 1$
2. $(a * b) + c = (b * c) + a$
3. $a * b = b * a$

Find the inverse of 5 under $*$. Record the absolute value of your answer.

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14. Find the remainder when $x^{100} - 4x^{98} + 5x + 6$ is divided by $x^3 - 2x^2 - x + 2$. Record the product of the coefficients.

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15. When is the first time after high noon (12 o'clock) that the second hand of an accurately set clock bisects the smaller angle formed by the hour hand and the minute hand? Find the number of seconds that have elapsed, express your number in reduced form $\frac{a}{b}$. Find $a + b$.