# CHMMC 2015 Tiebreaker Problems

November 22, 2015

**Problem 0.1.** *Call a positive integer $x$ n-cube-invariant if the last $n$ digits of $x$ are equal to the last $n$ digits of $x^3$. For example, 1 is n-cube invariant for any integer $n$. How many* 2015*-cube-invariant numbers $x$ are there such that $x < 10^{2015}$?*

**Problem 0.2.** *Let $a_1 = 1, a_2 = 1$, and for $n \geq 2$, let*

$$a_{n+1} = \frac{1}{n}a_n + a_{n-1}$$

*What is $a_{12}$?*

**Problem 0.3.** *Define an* n-digit pair cycle *to be a number with $n^2 + 1$ digits between 1 and $n$ with every possible pair of consecutive digits. For instance, 11221 is a 2-digit pair cycle since it contains the consecutive digits 11, 12, 22, and 21. How many 3-digit pair cycles exist?*

**Problem 0.4.** *The following number is the product of the divisors of $n$.*

$$46,656,000,000$$

*What is $n$?*