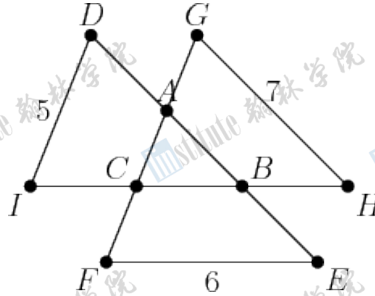


Caltech Harvey Mudd Mathematics Competition

Team Round

March 3, 2012

- Let a, b, c be positive integers. Suppose that $(a + b)(a + c) = 77$ and $(a + b)(b + c) = 56$. Find $(a + c)(b + c)$.
- In the diagram below, A and B trisect DE , C and A trisect FG , and B and C trisect HI . Given that $DI = 5$, $EF = 6$, $GH = 7$, find the area of $\triangle ABC$.



- In a 4×4 grid of sixteen unit squares, exactly 8 are shaded so that each shaded square shares an edge with exactly one other shaded square. How many ways can this be done?
- Let $P(x)$ be a monic polynomial of degree 3. Suppose that $P(x)$ has remainder $R(x)$ when it is divided by $(x - 1)(x - 4)$ and $2R(x)$ when it is divided by $(x - 2)(x - 3)$. Given that $P(0) = 5$, find $P(5)$.
- Suppose S is a subset of $\{1, 2, 3, 4, 5, 6, 7\}$. How many different possible values are there for the product of the elements in S ?
- Compute

$$\prod_{k=1}^{12} \left(\prod_{j=1}^{10} \left(e^{2\pi j i / 11} - e^{2\pi k i / 13} \right) \right).$$

(The notation $\prod_{k=a}^b f(k)$ means the product $f(a)f(a + 1) \cdots f(b)$.)

- A positive integer x is k -equivocal if there exists two positive integers b, b' such that when x is represented in base b and base b' , the two representations have digit sequences of length k that are permutations of each other. The smallest 2-equivocal number is 7, since 7 is 21 in base 3 and 12 in base 5. Find the smallest 3-equivocal number.
- A special kind of chess knight is in the origin of an infinite grid. It can make one of twelve different moves: it can move directly up, down, left, or right one unit square, or it can move 1 units in one direction and 3 units in an orthogonal direction. How many different squares can it be on after 2 moves?
- Let S be a square of side length 1, one of whose vertices is A . Let S^+ be the square obtained by rotating S clockwise about A by 30° . Let S^- be the square obtained by rotating S counterclockwise about A by 30° . Compute the total area that is covered by exactly two of the squares S, S^+, S^- . Express your answer in the form $a + b\sqrt{3}$ where a, b are rational numbers.
- A convex polygon in the Cartesian plane has all of its vertices on integer coordinates. One of the sides of the polygon is AB where $A = (0, 0)$ and $B = (51, 51)$, and the interior angles at A and B are both at most 45 degrees. Assuming no 180 degree angles, what is the maximum number of vertices this polygon can have?