Maritude Mar 14 18 182

Marith Mar H. & PR

Maritule Mar 14 18 198

Mariante Ant At '& PR

· 安子多常

Matinta Ar Ar 18 PR

Milling 素素 ·養 /家

Milital Market & PR

Y.

Y.

Y.

Y.

1

Y.

Y.

Y.

W.

Caltech Harvey Mudd Mathematics Competition

March 3, 2012

Maithe # * * *

Maithe # 14 18 PR

Maritute Aft At '8 PR

· *** ***

Maritud Mar 14 18 198

Inditute 新光·後序

Maritude 教教教·養學

Militalle Market 18 18

1. Let a_k be the number of ordered 10-tuples $(x_1, x_2, \dots x_{10})$ of nonnegative integers such that

$$x_1^2 + x_2^2 + \dots + x_{10}^2 = k.$$

Maritule Mark 18 182

Milling 新春 · 養 「寒

Maritute 新林·紫 序

Let $b_k = 0$ if a_k is even and $b_k = 1$ if a_k is odd. Find $\sum_{i=1}^{2012} b_{4i}$.

Maritud Mar No. 18 18.

Maritude Mar Har 18 180

Mininte 新林·蒙摩

Milling # 18 18

2. A convex octahedron in Cartesian space contains the origin in its interior. Two of its vertices are on the x-axis, two are on the y-axis, and two are on the z-axis. One triangular face F has side lengths $\sqrt{17}$, $\sqrt{37}$, $\sqrt{52}$. A second triangular face F' has side lengths $\sqrt{13}$, $\sqrt{29}$, $\sqrt{34}$. What is the minimum possible volume of the octahedron?

3. Three different faces of a regular dodecahedron are selected at random and painted. What is the probability that there is at least one pair of painted faces that share an edge?

4. The expression below has six empty boxes. Each box is to be filled in with a number from 1 to 6, where all six numbers are used exactly once, and then the expression is evaluated. What is the maximum possible final result that can be achieved?