Part 1

You might think this round is broken after solving some of these problems, but everything is intentional.

1. The number n can be represented uniquely as the sum of 6 distinct positive integers. Find n.

2. Let ABC be a triangle with $AB = BC$. The altitude from A intersects line BC at D. Suppose $BD = 5$ and $AC^2 = 1188$. Find AB.

3. A lemonade stand analyzes its earning and operations. For the previous month it had a \$45 dollar budget to divide between production and advertising. If it spent k dollars on production, it could make $2k - 12$ glasses of lemonade. If it spent k dollars on advertising, it could sell each glass at an average price of $15 + 5k$ cents. The amount it made in sales for the previous month was \$40.50. Assuming the stand spent its entire budget on production and advertising, what was the absolute difference between the amount spent on production and the amount spent on advertising?

4. Let A be the number of different ways to tile a $1 \times n$ rectangle with tiles of size 1×1, 1×3, and 1×6. Let B be the number of different ways to tile a $1 \times n$ rectangle with tiles of size 1×2 and 1×5, where there are 2 different colors available for the 1×2 tiles. Given that $A = B$, find n. (Two tilings that are rotations or reflections of each other are considered distinct.)

5. An integer $n \geq 0$ is such that n when represented in base 2 is written the same way as $2n$ is in base 5. Find n.

6. Let x be a positive integer such that $3, \log_6(12x), \log_6(18x)$ form an arithmetic progression in some order. Find x.

Part 2

Oops, it looks like there were some intentional printing errors and some of the numbers from these problems got removed. Any ■ that you see was originally some positive integer, but now its value is no longer readable. Still, if things behave like they did for Part 1, maybe you can piece the answers together.

1. The number n can be represented uniquely as the sum of ■ distinct positive integers. Find n.

2. Let ABC be a triangle with $AB = BC$. The altitude from A intersects line BC at D. Suppose $BD = $ ■ and $AC^2 = 1536$. Find AB.

3. A lemonade stand analyzes its earning and operations. For the previous month it had a \$50 dollar budget to divide between production and advertising. If it spent k dollars on production, it could make $2k - 2$ glasses of lemonade. If it spent k dollars on advertising, it could sell each glass at an average price of $25 + 5k$ cents. The amount it made in sales for the previous month was \$■. Assuming the stand spent its entire budget on production and advertising, what was the absolute difference between the amount spent on production and the amount spent on advertising?

4. Let A be the number of different ways to tile a $1 \times n$ rectangle with tiles of size $1 \times$ ■, $1 \times$ ■, and $1 \times$ ■. Let B be the number of different ways to tile a $1 \times n$ rectangle with tiles of size $1 \times$ ■ and $1 \times$ ■, where there are ■ different colors available for the $1 \times$ ■ tiles. Given that $A = B$, find n. (Two tilings that are rotations or reflections of each other are considered distinct.)

5. An integer $n \geq$ ■ is such that n when represented in base 9 is written the same way as $2n$ is in base ■. Find n.

6. Let x be a positive integer such that $1, \log_{96}(6x), \log_{96}($■$x)$ form an arithmetic progression in some order. Find x.