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资料 Individual Round: General II

February 9, 2019

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mistime ### Instructions

• DO NOT TURN OVER THIS PAPER UNTIL TOLD TO DO SO.

• This test contains 10 questions to be solved individually in 60 minutes.

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• All answers will be integers. • Opl-

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- Inte Mar H & Ph jull 新林 · 多 · R • Only answers written on the appropriate area on the answer sheet will be considered for grading.
- Problems are weighted relative to their difficulty, determined by the number of students who solve each problem.
- Similarly, graph paper, rulers, protractors, compasses, and other drawing aids are not permitted.
 If you believe the test contains and the second • No translators, books, notes, slide rules, calculators, abaci, or other computational aids are permitted.
 - If you believe the test contains an error, immediately tell your proctor and if necessary, report the error to the front desk after the end of your exam.

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• Good luck! 面的机机林塔张 而此此他新祥後

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- 1. Suppose x and y are one-digit positive integers such that $\frac{1}{x} = 0.\overline{9y}$ (i.e., $\frac{1}{x} = 0.9y9y9y$,...) and $\frac{1}{y} = 0.\overline{1x}$. What is x + y?
- 2. Consider three numbers, a, b, c, each of which is picked uniformly at random from the set $\{1, 2, 3, 4, 5\}$ (i.e. the integers between 1 and 9 inclusive). The probability that the quadratic equation $ax^2+bx+c=0$ has exactly two real roots can be expressed as a common fraction $\frac{m}{n}$. Find m+n.
- 3. Five distinct points are chosen inside or on a square of side length 4. Let m be the smallest possible number such that for any five given points, it is always possible to pick a pair of points from the five such that the two points are less than or equal to m units apart. We can write m in the form $\frac{a\sqrt{b}}{c}$, where \sqrt{b} is in simplest radical form and $\frac{a}{c}$ is a common fraction. What is a + b + c?
 - 4. The equation $2^{2x} 3^{2y} = 55$ has ordered pair solutions (x, y) where x and y are both integers. What is the sum of all x and y for all ordered pair solutions?
- 5. The infinite series $\frac{1}{10} + \frac{2}{100} + \frac{3}{1000} + \dots + \frac{n}{10^n} + \dots$ converges to *F*. Given that *F* can be expressed as a common fraction $\frac{a}{b}$, find a + b.
- 6. A set S of positive integers sum to 148. Repeats are allowed within this set. Let P be the largest possible product of all the integers in S. The prime factorization of P will have the form $\prod_{k=1}^{m} a_k^{b_k}$, where a_1, a_2, \ldots , and a_m are all of the distinct prime factors of P. What is the sum of all bases and exponents in the final product when expressed in this form?
 - 7. Two swimmers, starting from opposite ends of a 90 meter long pool, begin continuously swimming across the pool. One swimmer swims at the constant rate of 3 meters per second and the other swims at the constant rate of 2 meters per second. After swimming back and forth for 12 minutes, how many times did the two swimmers pass each other?
 - 8. Among all numbers x that satisfy $\sqrt[3]{x+9} \sqrt[3]{x-9} = 3$, find the largest possible value of x^2 .
 - 9. Right triangle $\triangle ABC$ has legs AC = 4 and BC = 3. Points M and N are drawn on hypotenuse \overline{AB} such that \overline{CM} and \overline{CN} trisect angle C. Given that the length of the shorter trisector can be written in the form $\frac{r\sqrt{s}-t}{w}$ where \sqrt{s} is in simplest radical form and the GCD of r, t, and w is 1, find r+s+t+w.

10. Nancy has a cube and five distinct colors. For each side of the cube, she chooses a color uniformly at random to paint that side of the cube. The probability that no two adjacent sides of the cube share the same color can be expressed as a common fraction $\frac{m}{n}$. Compute m + n.

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