Solutions

1. If $P(x) = a_2x^2 + a_1x + a_0$, then the coefficient sum is $a_2 + a_1 + a_0 = P(1) = \boxed{1}$.

2. We can see that if Kevin is one of the first three people in line, he will get a hotdog. If Kevin is 4th in line, he will have hotdog if less than two person bought two hotdogs: $(\frac{1}{2})$. If he is last, he will have a hotdog only if all people bought one hotdog. Thus the probability that he will not have a hotdog is $\frac{1}{5}(\frac{1}{2} + \frac{15}{16}) = \frac{23}{80}$. Thus our answer is $\boxed{103}$.

3. We can write an increasing arithmetic sequence of three terms as $a, a+b, a+2b$. We see that the smallest number and the largest number have same parity. Furthermore, we see that if the smallest and largest numbers are set, then the middle number is uniquely determined. Thus the answer is $2\binom{50}{2} = \boxed{2450}$

4. First, since $\lfloor x \rfloor = 12$, x must be between 144 and 169. Furthermore, $120 \le 10\sqrt{x} < 121$. Divide this inequality by 10 and square to yield $144 \le x < 146.21$. Thus, the probability is $\frac{2.41}{25} = \frac{241}{2500}$. The answer is thus $\boxed{2741}$.

5. Let the circumference of the track be a. Let the speed of Paul by x and speed of George be y. From the information that Paul and George meet when George ran 100 meters, we derive the equation $\frac{a}{2(x+y)} = \frac{100}{y}$. Furthermore, George meets Paul again after he ran 60 meters more than half the track. From this, we derive the equation $\frac{3a}{2(x+y)} = \frac{a+120}{2y}$. From these two equations, we see $\frac{a+120}{2y} = \frac{300}{y}$. thus $a = \boxed{480}$.

6. The prime factorization of the three numbers are $2^{10}5^{10}, 3^7 5^7, 2^{11}3^{22}$. We see that there are $(10+1)(10+1)$, $(7+1)(7+1)$, $(11+1)(22+1)$ divisors for each. We also see that these numbers have some identical divisors. Using the inclusion, exclusion theorem for three sets, we see that the answer is $(121 + 64 + 276 - (8 + 8 + 11) + 1) = \boxed{435}$.

7. First, observe that the numbers from 34 to 100 can all be included. Furthermore, the numbers from 1 to 11 can be included if 2 and 3 are left out. Thus, the largest possible set size is $(100-34+1)+(11-2) = \boxed{76}$.

8. There are $\binom{25}{3} = 2300$ total possibilities for choosing the three points, and 152 ways which are collinear (cf. OEIS A000938), so the result is $\frac{152}{2300} = \frac{38}{575}$, so $\boxed{613}$ is the answer.

9. Let I_j denote the indicator random variable for the event that original pair j are paired together for the dance. The sum of these indicators is the number of original couples that are paired together. Then by linearity of expectation, the desired value is:

$$2E\left(\sum_{j=1}^{18} I_j\right) = 2\sum_{j=1}^{18} E(I_j) = 36E(I_1) = 36P(\text{couple 1 paired together}) = \frac{36}{35}.$$

   The answer is therefore $36 + 35 = \boxed{71}$.

10. We can factor the given equation to $(x+y)(x-y) = a$. Because there needs to be four distinct pairs, a has to have at least 8 divisors. Furthermore, since $(x+y)$ and $(x-y)$ has the same parity, a should be factorized by two odd or two even numbers. If a is odd, all factors need to be odd. We see that the smallest possible number with eight divisor is $3 \times 5 \times 7 = 105$. If a is even, then factors of a must be written in the form $2x$ and $2y$, where $4xy = a$. xy must have 8 factors, and the smallest positive integer with 8 factors is 24. Thus $a = 96$, so the smallest value of a is $\boxed{96}$