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Individual Round: Calculus

February 9, 2019

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mistime ### Instructions

• DO NOT TURN OVER THIS PAPER UNTIL TOLD TO DO SO.

• This test contains 10 questions to be solved individually in 60 minutes.

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• All answers will be integers. • Opl-

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- 加北斯林省幣 jull 新林 · 多 · K • Only answers written on the appropriate area on the answer sheet will be considered for grading.
- Problems are weighted relative to their difficulty, determined by the number of students who solve each problem.
- Similarly, graph paper, rulers, protractors, compasses, and other drawing aids are not permitted.
 If you believe the test contains and the second • No translators, books, notes, slide rules, calculators, abaci, or other computational aids are permitted.
 - If you believe the test contains an error, immediately tell your proctor and if necessary, report the error to the front desk after the end of your exam.

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• Good luck! 而此此他就林塔路 面的机机林塔张

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Note: If necessary, recall that Euler's constant is $e \approx 2.718$. You will not need any more decimal places. itule the the $\int_{20}^{19} dx.$

1. Evaluate the definite integral



- 3. Determine the greatest integer less than or equal to $100\sum_{n=0}^{\infty} \frac{1}{(n+3)\cdot n!}.$
- 4. The value of the following series

$$\sum_{n=2}^{\infty} \frac{3n^2 + 3n + 1}{(n^2 + n)^3}$$

can be written in the form $\frac{m}{n}$, where m and n are coprime. Compute m + n

5. Given

$$4\int_{\ln 3}^{\ln 5} \frac{e^{3x}}{e^{2x} - 2e^x + 1} \, dx = a + b\ln 2,$$

where a and b are integers, what is the value of a + b?

6. The *double factorial* of a positive integer n is denoted n!! and equals the product $n(n-2)(n-4)\cdots(n-2(\lceil \frac{n}{2}\rceil - 1))$; we further specify that 0!! = 1. What is the greatest integer q such that

$$\sqrt[4]{q} < \sum_{n=0}^{\infty} \frac{1}{(2n)!!}?$$

7. Let e be Euler's constant. For all real x greater than e, let f(x) be the unique positive real value y satisfying y < x and $x^y = y^x$. Over $x \in (e, \infty)$, the function y = f(x) is differentiable, and the value of f'(4) can be expressed as $\frac{1}{a} - \frac{1}{b - \ln c}$ for positive integers a, b, and c. Compute the value of a + b + c.

- 8. A circle of radius 4 is tangent to the parabola $y = x^2$ at two distinct points and is centered at some point on the y-axis. The distance between the center of the circle and the origin (x, y) = (0, 0) can be expressed as $\frac{p}{q}$, for relatively prime positive integers p and q. Compute p + q.
- 9. A cylinder of radius 6 rests on the Euclidean plane, with the center of its base at the origin. One end of a string of length 6π is attached to the cylinder at the point (6, 0). is negligible. The area of the region on the plane that can be reached by the free end of the string can be written as $m\pi^3$ for a natural number m. Find the value of m.
- 10. In the Euclidean plane, vertices A(-1,0), B(1,0), and C(x,y) form a triangle with perimeter 12. What inte to the second second Arithte # *** withte the the 's is the largest possible integer value of x + y? stitute # #

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