1. Alice is planning a trip from the Bay Area to one of 5 possible destinations (each of which is serviced by only 1 airport) and wants to book two flights, one to her destination and one returning. There are 3 airports within the Bay Area from which she may leave and to which she may return. In how many ways may she plan her flight itinerary?

2. Determine the largest integer n such that 2^n divides the decimal representation given by some permutation of the digits 2, 0, 1, and 5. (For example, 2^1 divides 2150. It may start with 0.)

3. How many rational solutions are there to $5x^2 + 2y^2 = 1$?

4. Determine the greatest integer N such that N is a divisor of $n^{13} - n$ for all integers n.

5. Three balloon vendors each offer two types of balloons − one offers red & blue, one offers blue & yellow, and one offers yellow & red. I like each vendor the same, so I must buy 7 balloons from each. How many different possible triples (x, y, z) are there such that I could buy x blue, y yellow, and z red balloons?

6. There are 30 cities in the empire of Euleria. Every week, Martingale City runs a very well-known lottery. 900 visitors decide to take a trip around the empire, visiting a different city each week in some random order. 3 of these cities are inhabited by mathematicians, who will talk to all visitors about the laws of statistics. A visitor with this knowledge has probability 0 of buying a lottery ticket, else they have probability 0.5 of buying one. What is the expected number of visitors who will play the Martingale Lottery?

7. At Durant University, an A grade corresponds to raw scores between 90 and 100, and a B grade corresponds to raw scores between 80 and 90. Travis has 3 equally weighted exams in his math class. Given that Travis earned an A on his first exam and a B on his second (but doesnt know his raw score for either), what is the minimum score he needs to have a 90% chance of getting an A in the class? Note that scores on exams do not necessarily have to be integers.

8. Two players play a game with a pile with N coins is on a table. On a player's turn, if there are n coins, the player can take at most $n/2 + 1$ coins, and must take at least one coin. The player who grabs the last coin wins. For how many values of N between 1 and 100 (inclusive) does the first player have a winning strategy?

9. There exists a unique pair of positive integers k, n such that k is divisible by 6, and $\sum_{i=1}^{k} i^2 = n^2$. Find (k, n).

10. A partition of a positive integer n is a summing $n_1 + ... + n_k = n$, where $n_1 \geq n_2 \geq ... \geq n_k$. Call a partition *perfect* if every $m \leq n$ can be represented uniquely as a sum of some subset of the n_i's. How many perfect partitions are there of $n = 307$?

P1. Find two disjoint sets N_1 and N_2 with $N_1 \cup N_2 = \mathbb{N}$, so that neither set contains an infinite arithmetic progression.

P2. Suppose $k > 3$ is a divisor of $2^p + 1$, where p is prime. Prove that $k \geq 2p + 1$.