

1. Let x, y, z, w be integers such that $2^x + 2^y + 2^z + 2^w = 24.375$. Find the value of $xyzw$.
2. Let $g(x) = 1 + 2x + 3x^2 + 4x^3 + \dots$. Find the coefficient of x^{2015} of $f(x) = \frac{g(x)}{1-x}$.
3. Find all integer solutions to

$$\begin{aligned}x^2 + 2y^2 + 3z^2 &= 36, \\3x^2 + 2y^2 + z^2 &= 84, \\xy + xz + yz &= -7.\end{aligned}$$

4. Let $\{a_n\}$ be a sequence of real numbers with $a_1 = -1, a_2 = 2$ and for all $n \geq 3$,

$$a_{n+1} - a_n - a_{n+2} = 0.$$

Find $a_1 + a_2 + a_3 + \dots + a_{2015}$.

5. Let x and y be real numbers satisfying the equation $x^2 - 4x + y^2 + 3 = 0$. If the maximum and minimum values of $x^2 + y^2$ are M and m respectively, compute the numerical value of $M - m$.
6. The roots of the equation $x^5 - 180x^4 + Ax^3 + Bx^2 + Cx + D = 0$ are in geometric progression. The sum of their reciprocals is 20. Compute $|D|$.
7. Evaluate $\sum_{k=0}^{37} (-1)^k \binom{75}{2k}$
8. Let ω be a primitive 7th root of unity. Find

$$\prod_{k=0}^6 (1 + \omega^k - \omega^{2k}).$$

(A complex number is a primitive root of unity if and only if it can be written in the form $e^{2k\pi i/n}$, where k is relatively prime to n .)

9. Find

$$\lim_{n \rightarrow \infty} \frac{1}{n^3} \left(\sqrt{n^2 - 1} + \sqrt{n^2 - 2^2} + \dots + \sqrt{n^2 - (n-1)^2} \right).$$

10. Evaluate

$$\int_0^{\pi/2} \ln(4 \sin x) dx.$$

- P1.** Suppose z_0, z_1, \dots, z_{n-1} are complex numbers such that $z_k = e^{2k\pi i/n}$ for $k = 0, 1, 2, \dots, n-1$.

Prove that for any complex number z , $\sum_{k=0}^{n-1} |z - z_k| \geq n$.

- P2.** Let $f(x)$ be a nonconstant monic polynomial of degree n with rational coefficients that is irreducible, meaning it cannot be factored into two nonconstant rational polynomials. Find and prove a formula for the number of monic complex polynomials that divide f .