Time limit: 40 minutes.
Instructions: This test contains 15 short answer questions. All answers must be expressed in simplest form unless specified otherwise. Only answers written inside the boxes on the answer sheet will be considered for grading.
No calculators.

1. Find the maximum integral value of k such that $0 \le k \le 2019$ and

$$|e^{2\pi i \frac{k}{2019}} - 1|$$

   is maximal.

2. Find the remainder when 2^{2019} is divided by 7.

3. A cylinder with radius 5 and height 1 is rolling on the (unslanted) floor. Inside the cylinder, there is water that has constant height $\dfrac{15}{2}$ as the cylinder rolls on the floor. What is the volume of the water?

4. Let C be the number of ways to arrange the letters of the word CATALYSIS, T be the number of ways to arrange the letters of the word TRANSPORT, S be the number of ways to arrange the letters of the word STRUCTURE, and M be the number of ways to arrange the letters of the word MOTION. What is $\dfrac{C - T + S}{M}$?

5. What is the minimum distance between $(2019, 470)$ and $(21a - 19b, 19b + 21a)$ for $a, b \in \mathbb{Z}$?

6. At a party, 2019 people decide to form teams of three. To do so, each turn, every person not on a team points to one other person at random. If three people point to each other (that is, A points to B, B points to C, and C points to A), then they form a team. What is the probability that after $65,536$ turns, exactly one person is not on a team?

7. How many distinct ordered pairs of integers (b, m, t) satisfy the equation $b^8 + m^4 + t^2 + 1 = 2019$?

8. Let (k_i) be a sequence of unique nonzero integers such that $x^2 - 5x + k_i$ has rational solutions. Find the minimum possible value of

$$\frac{1}{5} \sum_{i=1}^{\infty} \frac{1}{k_i}.$$

9. You wish to color every vertex, edge, face, and the interior of a cube one color each such that no two adjacent objects are the same color. Faces are adjacent if they share an edge. Edges are adjacent if they share a vertex. The interior is adjacent to all of its faces, edges, and vertices. Each face is adjacent to all of its edges and vertices. Each edge is adjacent to both of its vertices. What is the minimum number of colors required to do this?

10. Compute the remainder when the product of all positive integers less than and relatively prime to 2019 is divided by 2019.

11. A baseball league has 64 people, each with a different 6-digit binary number whose base-10 value ranges from 0 to 63. When any player bats, they do the following: for each pitch, they swing if their corresponding bit number is a 1; otherwise, they decide to wait and let the ball pass. For example, the player with the number 11 has binary number 001011. For the first and second

pitch, they wait; for the third, they swing, and so on. Pitchers follow a similar rule to decide whether to throw a splitter or a fastball; if the bit is 0, they will throw a splitter, and if the bit is 1, they will throw a fastball.

If a batter swings at a fastball, then they will score a hit; if they swing on a splitter, they will miss and get a "strike." If a batter waits on a fastball, then they will also get a strike. If a batter waits on a splitter, then they get a "ball." If a batter gets 3 strikes, then they are out; if a batter gets 4 balls, then they automatically get a hit. For example, if player 11 pitched against player 6 (binary is 000110), the batter would get a ball for the first pitch, a ball for the second pitch, a strike for the third pitch, a strike for the fourth pitch, and a hit for the fifth pitch; as a result, they will count that as a "hit." If player 11 pitched against player 5 (binary is 000101), however, then the fifth pitch would be the batter's third strike, so the batter would be "out."

Each player in the league plays against every other player exactly twice; once as batter, and once as pitcher. They are then given a score equal to the number of outs they throw as a pitcher plus the number of hits they get as a batter. What is the highest score received?

12. 2019 people (all of whom are perfect logicians), labeled from 1 to 2019, partake in a paintball duel. First, they decide to stand in a circle, in order, so that Person 1 has Person 2 to his left and person 2019 to his right. Then, starting with Person 1 and moving to the left, every person who has not been eliminated takes a turn shooting. On their turn, each person can choose to either shoot one non-eliminated person of his or her choice (which eliminates that person from the game), or deliberately miss. The last person standing wins. If, at any point, play goes around the circle once with no one getting eliminated (that is, if all the people playing decide to miss), then automatic paint sprayers will turn on, and end the game with everyone losing. Each person will, on his or her turn, always pick a move that leads to a win if possible, and, if there is still a choice in what move to make, will prefer shooting over missing, and shooting a person closer to his or her left over shooting someone farther from their left. What is the number of the person who wins this game? Put "0" if no one wins.

13. Triangle $\triangle ABC$ has $AB = 13$, $BC = 14$, and $CA = 15$. $\triangle ABC$ has incircle γ and circumcircle ω. γ has center at I. Line $\overline{AI}$ is extended to hit ω at P. What is the area of quadrilateral $ABPC$?

14. A regular hexagon has positive integer side length. A laser is emitted from one of the hexagon's corners, and is reflected off the edges of the hexagon until it hits another corner. Let a be the distance that the laser travels. What is the smallest possible value of a^2 such that $a > 2019$? You need not simplify/compute exponents.

15. A group of aliens from Gliese 667 Cc come to Earth to test the hypothesis that mathematics is indeed a universal language. To do this, they give you the following information about their mathematical system:

- For the purposes of this experiment, the Gliesians have decided to write their equations in the same syntactic format as in Western math. For example, in Western math, the expression "5+4" is interpreted as running the "+" operation on numbers 5 and 4. Similarly, in Gliesian math, the expression $\alpha\gamma\beta$ is interpreted as running the "γ" operation on numbers α and β.

- You know that γ and η are the symbols for addition and multiplication (which works the same in Gliesian math as in Western math), but you don't know which is which. By some

bizarre coincidence, the symbol for equality is the same in Gliesian math as it is in Western math; equality is denoted with an "=" symbol between the two equal values.

- Two symbols that look exactly the same have the same meaning. Two symbols that are different have different meanings and, therefore, are not equal.

They then provide you with the following equations, written in Gliesian, which are known to be true:

$$\Cap \eta \rhd = \curlywedge \qquad \odot \gamma \varkappa = \gg \qquad \ltimes \gamma \gg = \curlywedge$$
$$\gg \gamma \lozenge = \rhd \qquad \gg \eta \Cup = \lozenge \qquad \gg \eta \ltimes = \varkappa$$
$$\Box \gamma \gg = \Cap \qquad \Cap \eta \Cup = \Box \qquad \Box \eta \Cup = \gg$$

What is the human number equivalent of $\odot$?