Time limit: 60 minutes.

**Instructions:** This test contains 10 short answer questions. All answers must be expressed in simplest form unless specified otherwise. Only answers written inside the boxes on the answer sheet will be considered for grading.

## No calculators.

1. Consider the figure below, where every small triangle is equilateral with side length 1. Compute the area of the polygon AEKS.



- 2. A set of points in the plane is called full if every triple of points in the set are the vertices of a non-obtuse triangle. What is the largest size of a full set?
- 3. Let ABCD be a parallelogram with BC = 17. Let M be the midpoint of BC and let N be the point such that DANM is a parallelogram. What is the length of segment NC?



- 4. The area of right triangle ABC is 4, and hypotenuse AB is 12. Compute the perimeter of ABC.
- 5. Find the area of the set of all points z in the complex plane that satisfy

$$|z - 3i| + |z - 4| \le 5\sqrt{2}.$$

- 6. Let ABE be a triangle with AB/3 = BE/4 = EA/5. Let  $D \neq A$  be on line AE such that AE = ED and D is closer to E than to A. Moreover, let C be a point such that BCDE is a parallelogram. Furthermore, let M be on line CD such that AM bisects  $\angle BAE$ , and let P be the intersection of AM and BE. Compute the ratio of PM to the perimeter of ABE.
- 7. Points ABCD are vertices of an isosceles trapezoid, with AB parallel to CD, AB = 1, CD = 2, and BC = 1. Point E is chosen uniformly and at random on CD, and let point F be the point on CD such that EC = FD. Let G denote the intersection of AE and BF, not necessarily in the trapezoid. What is the probability that  $\angle AGB > 30^{\circ}$ ?



- 8. Let ABC be a triangle with AB = 13, BC = 14, and CA = 15. Let G denote the centroid of ABC, and let  $G_A$  denote the image of G under a reflection across BC, with  $G_B$  the image of G under a reflection across AB. Let  $O_G$  be the circumcenter of  $G_A G_B G_C$  and let X be the intersection of  $AO_G$  with BC and Y denote the intersections of AG with BC. Compute XY.
- 9. Let ABCD be a tetrahedron with  $\angle ABC = \angle ABD = \angle CBD = 90^{\circ}$  and AB = BC. Let E, F, G be points on AD, BD, and CD, respectively, such that each of the quadrilaterals AEFB, BFGC, and CGEA have an inscribed circle. Let r be the smallest real number such that area $(EFG)/\text{area}(ABC) \leq r$  for all such configurations A, B, C, D, E, F, G. If r can be expressed as  $\frac{\sqrt{a-b}\sqrt{c}}{d}$  where a, b, c, d are positive integers with gcd(a, b) squarefree and c squarefree, find a + b + c + d.
- 10. A 3-4-5 point of a triangle *ABC* is a point *P* such that the ratio *AP* : *BP* : *CP* is equivalent to the ratio 3:4:5. If *ABC* is isosceles with base BC = 12 and *ABC* has exactly one 3-4-5 point, compute the area of *ABC*.