

1. Compute the probability that a random permutation of the letters in BERKELEY does not have the three E's all on the same side of the Y.

Answer: $\frac{1}{2}$ or 0.5

Solution: In total, there are $\frac{8!}{3!} = 56 \cdot 120$ permutations. Of these, if the Y is in slots 1-3 (from the left), or slots 6-8 (from the left), then the 3 E's must be to the right or to the left of the Y, respectively. By symmetry, there are $2 \cdot \left(\binom{7}{3} + \binom{6}{3} + \binom{5}{3} \right) \cdot 4! = 48 \cdot (35 + 20 + 10) = 48 \cdot 65$ such permutations. In addition, the Y can also be in the middlemost two slots, in which case the E's can be either to the left or to the right of it. WLOG assume Y is in slot 4. There are $1 \cdot 4! = 24$ permutations with the E's to the left and $\binom{4}{3} \cdot 4! = 96$ with the E's to the right (since there are 4 spaces to the right of the Y), which yields 120 permutations in total in this case; multiplying by 2 yields an additional 240 for when the Y is in slot 4 or 5. Note that $240 = 48 \cdot 5$, so in total, there are $48 \cdot 70$ invalid permutations out of $56 \cdot 120$ in total. This yields a probability that a permutation has all 3 E's on the same side as the Y of $\frac{48 \cdot 70}{56 \cdot 120} = \frac{48}{120} \cdot \frac{70}{56} = \frac{2}{5} \cdot \frac{5}{4} = \frac{1}{2}$, so by complementary counting, the probability is also $\boxed{\frac{1}{2}}$ that a permutation does *not* have all 3 E's on the same side of Y.

2. Find the sum of first two integers $n > 1$ such that 3^n is divisible by n and $3^n - 1$ is divisible by $n - 1$.

Answer: 30

Solution: Since 3^n is divisible n , we know that n must be a power of three. Let $n = 3^a$. Then we have $3^a - 1 | 3^n - 1$, and using division theorem and simple algebraic manipulations, we can see that a must divide n . Therefore, the solution is in the form $n = 3^{3^a}$, where a is any nonnegative integer. The first two such n are $3^{3^0} = 3$ and $3^{3^1} = 27$, and the sum is just $\boxed{30}$.

3. Let $\{a, b, c, d, e, f, g, h\}$ be a permutation of $\{1, 2, 3, 4, 5, 6, 7, 8\}$. What is the probability that $\overline{abc} + \overline{def}$ is even?

Answer: $\frac{3}{7}$

Solution: Both c and f have to have the same parity so the total number of valid arrangements is $2 \cdot \binom{4}{2} \cdot 6!$, and the total number of permutations is $8!$. Thus, the answer is $\boxed{\frac{3}{7}}$