

1. Compute the maximum real value of  $a$  for which there is an integer  $b$  such that  $\frac{ab^2}{a+2b} = 2019$ . Compute the maximum possible value of  $a$ .

**Answer: 30285**

**Solution:** We can solve the equation for  $a$ , obtaining  $a = \frac{4038b}{b^2-2019}$ . This is maximized when  $b^2$  is as close to 2019 while still exceeding it. We let  $b = 45$  and as a result  $a = 30285$ .

2. If  $P$  is a function such that  $P(2x) = 2^{-3}P(x) + 1$ , find  $P(0)$ .

**Answer:  $\frac{8}{7}$  or  $1\frac{1}{7}$**

**Solution:** Plugging in  $x = 0$ , we obtain

$$P(0)(1 - 2^{-3}) = 1,$$

so  $P(0) = \boxed{\frac{8}{7}}$ .

3. There are two equilateral triangles with a vertex at  $(0, 1)$ , with another vertex on the line  $y = x + 1$  and with the final vertex on the parabola  $y = x^2 + 1$ . Find the area of the larger of the two triangles.

**Answer:  $45 + 26\sqrt{3}$**

**Solution:** We can shift all three vertices down one unit with no change to the areas of the triangle. We then have a vertex at  $(0, 0)$ , another vertex at  $(a, a)$ , and a third vertex at  $(b, b^2)$ . Representing as polar coordinates, they are at  $0$ ,  $(a + ai)$ , and  $b + b^2i$ . We know that either  $(a + ai)(\frac{1}{2} + \frac{\sqrt{3}}{2}i) = b + b^2i$ , or  $(b + b^2i)(\frac{1}{2} + \frac{\sqrt{3}}{2}i) = a + ai$ . Solving both, we see that our maximal triangle has solution where  $a = 5 + 3\sqrt{3}$  (which comes from the first equation; the second equation only has the solution  $a = b = 0$ ), which gives us an area of  $45 + 26\sqrt{3}$ .