Time limit: 60 minutes.
Instructions: This test contains 10 short answer questions. All answers must be expressed in simplest form unless specified otherwise. Only answers written inside the boxes on the answer sheet will be considered for grading.
No calculators.

1. How many multiples of 20 are also divisors of 17!?

2. Suppose for some positive integers, that $\frac{p+\frac{1}{q}}{q+\frac{1}{p}} = 17$. What is the greatest integer n such that $\frac{p+q}{n}$ is always an integer?

3. Find the minimal N such that any N-element subset of $\{1, 2, 3, 4, \ldots 7\}$ has a subset S such that the sum of elements of S is divisible by 7.

4. What is the remainder when 201820182018... [2018 times] is divided by 15?

5. If r_i are integers such that $0 \leq r_i < 31$ and r_i satisfies the polynomial $x^4 + x^3 + x^2 + x \equiv 30 \pmod{31}$, find
$$\sum_{i=1}^{4} (r_i^2 + 1)^{-1} \pmod{31}$$
where x^{-1} is the modulo inverse of x, that is, it is the unique integer y such that $0 < y < 31$ and $xy - 1$ is divisible by 31.

6. Ankit wants to create a pseudo-random number generator using modular arithmetic. To do so he starts with a seed x_0 and a function $f(x) = 2x + 25 \pmod{31}$. To compute the kth pseudo random number, he calls $g(k)$ defined as follows:
$$g(k) = \begin{cases} x_0 & \text{if } k = 0 \\ f(g(k-1)) & \text{if } k > 0 \end{cases}$$
If x_0 is 2017, compute $\sum_{j=0}^{2017} g(j) \pmod{31}$.

7. Determine the number of ordered triples (a, b, c), with $0 \leq a, b, c \leq 10$ for which there exists (x, y) such that $ax^2 + by^2 \equiv c \pmod{11}$

8. How many $1 < n \leq 2018$ such that the set $\{0, 1, 1+2, \ldots, 1+2+3+\cdots+i, \ldots, 1+2+\cdots+n-1\}$ is a permutation of $\{0, 1, 2, 3, 4, \cdots, n-1\}$ when reduced modulo n?

9. Compute the following:
$$\sum_{x=0}^{99} (x^2 + 1)^{-1} \pmod{199}$$
where x^{-1} is the value $0 \leq y \leq 199$ such that $xy - 1$ is divisible by 199.

10. Evaluate the following
$$\prod_{j=1}^{50} \left(2\cos\left(\frac{4\pi j}{101}\right) + 1 \right)$$