Time limit: 60 minutes.
Instructions: This test contains 10 short answer questions. All answers must be expressed in simplest form unless specified otherwise. Only answers written inside the boxes on the answer sheet will be considered for grading.
No calculators.

1. You have 9 colors of socks and 5 socks of each type of color. Pick two socks randomly. What is the probability that they are the same color?

2. Each BMT, every student chooses one of three focus rounds to take. Bob plans to attend BMT for the next 4 years and wants to figure out what focus round to take each year. Given that he wants to take each focus round at least once, how many ways can he choose which round to take each year?

3. What is the smallest positive integer with exactly 7 distinct proper divisors?

4. What is the greatest multiple of 9 that can be formed by using each of the digits in the set $\{1, 3, 5, 7, 9\}$ at most once.

5. How many subsets of $\{1, 2, \ldots, 9\}$ do not contain 2 adjacent numbers?

6. Let $S = \{1, 2, \ldots 6\}$. How many functions $f : S \to S$ are there such that for all $s \in S$,
$$f^5(s) = f(f(f(f(f(s))))) = 1.$$

7. A light has been placed on every lattice point (point with integer coordinates) on the (infinite) 2D plane. Define the Chebyshev distance between points (x_1, y_1) and (x_2, y_2) to be $\max(|x_1 - x_2|, |y_1 - y_2|)$. Each light is turned on with probability $\frac{1}{2^{d/2}}$, where d is the Chebyshev distance from that point to the origin. What is expected number of lights that have all their directly adjacent lights turned on? (Adjacent points being points such that $|x_1 - x_2| + |y_1 - y_2| = 1$.)

8. In a 1024 person randomly seeded single elimination tournament bracket, each player has a unique skill rating. In any given match, the player with the higher rating has a $\frac{3}{4}$ chance of winning the match. What is the probability the **second** lowest rated player wins the tournament?

9. n balls are placed independently uniformly at random into n boxes. One box is selected at random, and is found to contain b balls. Let e_n be the expected value of b^4. Find
$$\lim_{n \to \infty} e_n.$$

10. Let $\phi(n)$ be the number of positive integers less than or equal to n that are relatively prime to n. Evaluate
$$\sum_{n=1}^{64} (-1)^n \phi(n) \left\lfloor \frac{64}{n} \right\rfloor.$$