Time limit: 60 minutes.
Instructions: This test contains 10 short answer questions. All answers must be expressed in simplest form unless specified otherwise. Only answers written inside the boxes on the answer sheet will be considered for grading.
No calculators.

1. 10 students take the Analysis Round. The average score was a 3 and the high score was a 7. If no one got a 0, what is the maximum number of students that could have achieved the high score?

2. Find all solutions to $3^x - 9^{x-1} = 2$.

3. Compute
$$\int_{-9}^{9} 17x^3 \cos(x^2)\, dx.$$

4. Find the value of
$$\frac{1}{2} + \frac{4}{2^2} + \frac{9}{2^3} + \frac{16}{2^4} + \cdots .$$

5. Find the value of y such that the following equation has exactly three solutions.
$$||x-1|-4| = y.$$

6. Consider the function $f(x, y, z) = (x - y + z, y - z + x, z - x + y)$ and denote by $f^{(n)}(x, y, z)$ the function f applied n times to the tuple (x, y, z). Let r_1, r_2, r_3 be the three roots of the equation $x^3 - 4x^2 + 12 = 0$ and let $g(x) = x^3 + a_2x^2 + a_1x + a_0$ be the cubic polynomial with the tuple $f^{(3)}(r_1, r_2, r_3)$ as roots. Find the value of a_1.

7. Compute
$$\sum_{k=1}^{\infty} \frac{(-1)^k}{(2k-1)(2k+1)}.$$

8. The numerical value of the following integral
$$\int_0^1 (-x^2 + x)^{2017} \lfloor 2017x \rfloor \, dx$$
can be expressed in the form $a\frac{m!^2}{n!}$ where a is minimized. Find $a + m + n$. (Note $\lfloor x \rfloor$ is the largest integer less than or equal to x.)

9. Let a_d be the number of non-negative integer solutions (a, b) to $a + b = d$ where $a \equiv b \pmod{n}$ for a fixed $n \in \mathbb{Z}^+$. Consider the generating function $M(t) = a_0 + a_1t + a_2t^2 + \ldots$. Consider
$$P(n) = \lim_{t \to 1} \left(nM(t) - \frac{1}{(1-t)^2} \right).$$
Then $P(n), n \in \mathbb{Z}^+$ is a polynomial in n, so we can extend its domain to include all real numbers while having it remain a polynomial. Find $P(0)$.

10. Define $H_n = \sum_{k=1}^{n} \frac{1}{k}$. Evaluate
$$\sum_{n=1}^{2017} \binom{2017}{n} H_n (-1)^n.$$