

1. A $2 \times 4 \times 8$ rectangular prism and a cube have the same volume. What is the difference between their surface areas?
2. Cyclic quadrilateral $ABCD$ has side lengths $AB = 6, BC = 7, CD = 7, DA = 6$. What is the area of $ABCD$?
3. Let S be the set of all non-degenerate triangles with integer sidelengths, such that two of the sides are 20 and 16. Suppose we pick a triangle, at random, from this set. What is the probability that it is acute?
4. ABC is an equilateral triangle, and $ADEF$ is a square. If D lies on side AB and E lies on side BC , what is the ratio of the area of the equilateral triangle to the area of the square?
5. Convex pentagon $ABCDE$ has the property that $\angle ADB = 20^\circ$, $\angle BEC = 16^\circ$, $\angle CAD = 3^\circ$, and $\angle DBE = 12^\circ$. What is the measure of $\angle ECA$?
6. Triangle ABC has sidelengths $AB = 13, AC = 14$, and $BC = 15$ and centroid G . What is the area of the triangle with sidelengths AG, BG , and CG ?
7. Let ABC be a right triangle with $AB = BC = 2$. Construct point D such that $\angle DAC = 30^\circ$ and $\angle DCA = 60^\circ$, and $\angle BCD > 90^\circ$. Compute the area of triangle BCD .
8. A regular unit 7-simplex is a polytope in 7-dimensional space with 8 vertices that are all exactly a distance of 1 apart. (It is the 7-dimensional analogue to the triangle and the tetrahedron.) In this 7-dimensional space, there exists a point that is equidistant from all 8 vertices, at a distance d . Determine d .
9. Given right triangle ABC with right angle at C , construct three external squares $ABDE$, $BCFG$, and $ACHI$. If $DG = 19$ and $EI = 22$, compute the length of FH .
10. Triangle ABC has side lengths $AB = 5, BC = 9$, and $AC = 6$. Define the incircle of ABC to be C_1 . Then, define C_i for $i > 1$ to be externally tangent to C_{i-1} and tangent to AB and BC . Compute the sum of the areas of all circles C_n .