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1. 130°

2. $\frac{1}{3}$

3. $\frac{27}{2}$

4. 13

5. 2

6. $\frac{4}{15}$

7. $\frac{10}{3}$

8. $\left(-\frac{\sqrt{3}}{2}, 1\right)$

9. $\frac{9}{10}$

10. $\frac{8}{5}$

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P1. $\angle BXB' = \angle AXA' = \angle AYA' = \angle CYC'$. Thus arcs BB' and CC' have the subtend the same angle in circle C_2, so the corresponding segments are congruent.

- 1 point for a reasonably good diagram.
- 1 point for $\angle XAY = \angle XA'Y$.
- 2 points for relating angles in C_1 with angles in C_2
- 2 points for concluding that equal segments follow from equal angles.

P2. **Solution 1:** Suppose that $A = (0,0)$, l is $y = 0$, and the center of C_1 is $(0,a)$. Suppose that C_2 has a radius r and center (x,y). By tangency with l, $y = r$. By tangency with C_1, $x^2 + (y-a)^2 = (r+a)^2 \iff 4ya = x^2$. Thus it is necessary any point in the locus lies on the parabola $y = \frac{x^2}{4a}$. If we start with any point on the parabola, we can construct such a tangent circle with radius $r = y$ as long as $x \neq 0$, which results in a circle of zero radius where $B = A$. Thus the locus is $\{(x,y) : x \neq 0 \,\text{and}\, y = \frac{x^2}{4a}\}$. I.e. a parabola minus a point.
Solution 2: Note that a parabola is the locus of all points that are equidistant from a line and a point. Let the point be the center of C_1 and the line l_2 be a distance a away from line l (away from the circle). Any center of C_2 is a distance $r_1 + r_2$ away from l_2 as well as O_1. The excluded point is excluded because that would involve a circle of radius 0.

- 1 point for using the distance formula to set up the formula.
- 2 points for simplifying the formula into a parabola
- 2 points for the reverse argument (showing that a point on the parabola has a circle)
- 1 point for addressing the one point that is removed from the parabola

Solution 2 Rubric:

- 3 points for correctly defining the locus of the points defining a parabola
- 1 point for computing the distance from the center of C_1 to the centers of C_2
- 1 point for computing the distance from the center of C_1 to line l_2
- 1 point for addressing the one point that is removed from the parabola