1. Note that $2^{-2} + 2^{-3} = \frac{1}{4} + \frac{1}{8} = 0.375$. In addition, $2^4 + 2^3 = 16 + 8 = 24$. Hence the final answer is $(-2)(-3)(4)(3) = \boxed{72}$

2. We have that

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + x^4 \dots$$
$$\left(\frac{1}{1-x}\right)' = \frac{1}{(1-x)^2} = 1 + 2x + 3x^2 + 4x^3 + \dots = g(x)$$
$$g(x) = \sum_{k=0}^{\infty}(k+1)x^k$$

Differentiating again we have

$$g(x)' = \sum_{k=0}^{\infty}(k+1)kx^{k-1} = \left(\frac{1}{(1-x)^2}\right)' = \frac{2}{(1-x)^3} = \frac{2g(x)}{1-x} = 2f(x)$$
$$\sum_{k=0}^{\infty}(k+1)kx^{k-1} = 2f(x)$$
$$\sum_{k=0}^{\infty}\frac{(k+1)k}{2}x^{k-1} = f(x)$$

Hence the coefficent of x^{2015} will be $\frac{(2017)(2016)}{2} = \boxed{\binom{2017}{2}}$

3. We add the first two equations to get:

$$x^2 + 2y^2 + 3z^2 = 36 \quad (1)$$
$$+3x^2 + 2y^2 + z^2 = 84 \quad (2)$$
$$4x^2 + 4y^2 + 4z^2 = 120 \quad (3)$$
$$x^2 + y^2 + z^2 = 30 \quad (4)$$

We can now put the above equation into equation 1 and 2 to get $x^2 - z^2 = 24$. Now adding two times equation 4 and two times $xy + xz + yz$ we have:

$$2(x^2 + y^2 + z^2) + 2(xy + xz + yz) =$$
$$(x+z)^2 + (x+y)^2 + (y+z)^2 = 2(30) + 2(-7) = 46.$$

From here it is easy to check for integer solutions by plugging in the squares from 1 to 6 and then checking whether the remaining number can be expressed as the sum of two squares. (It might be helpful to know that a number can be expressed as a sum of two squares if and only if its factorization into distinct primes contains no odd powers of primes congruent to 3 modulo 4). We then see that the only solution that works is $6^2 + 3^2 + 1^1 = 36 + 9 + 1 = 46$. Combining this with $x^2 - z^2 = 24$ we get that the only integer solutions are $\boxed{(5, -2, 1), (-5, 2, -1)}$

4. From the recurrence relation we have $a_n = a_{n+1} - a_{n+2} = a_{n+1} - (a_{n+1} + a_{n+3}) = -a_{n+3} = a_{n+6}$. Hence the sequence cycles with period 6. Writing out the first few terms and noting that $2015 \equiv 5 \mod 6$, we get that $a_1 + a_2 + a_3 + \dots + a_{2015} = a_1 + a_2 + a_3 + a_4 + a_5 = -1 + 2 + 3 + 1 - 2 = \boxed{3}$

5. Completing the square we get $x^2-4x+y^2+3=(x-2)^2+y^2=1$. From here we see that x^2+y^2 would be the square of distance from the orgin to a point on the circle $(x-2)^2+y^2=1$. The maximum and minimum distance would then be 3 and 1 respectively so our answer is $9-1=\boxed{8}$

6. Let the five roots be $a, ar, ar^2, ar^3, ar^4, ar^5$. We are then given that:

$$a(1+r+r^2+r^3+r^4)=180 \tag{5}$$
$$-a^5r^{10}=D \tag{6}$$
$$\frac{1}{a}+\frac{1}{ar}+\frac{1}{ar^2}+\frac{1}{ar^3}+\frac{1}{ar^4}=20 \tag{7}$$

from Vieta's formulas. Now simplifying equation 7 we have:

$$\begin{aligned}\frac{1+r+r^2+r^3+r^4}{ar^4}&=20\\ \frac{180}{a^2r^4}&=20\\ (ar^2)^2&=9\\ ar^2&=\pm 3\end{aligned}$$

Hence $D=-(ar^2)^5=-(\pm 3)^5=\pm 243 \implies |D|=\boxed{243}$

7. By the binomial theorem we have $(1+x)^{75}=\sum_{k=0}^{75}\binom{75}{k}$ Plugging in i we see that that our sum $S=\sum_{k=0}^{37}(-1)^k\binom{75}{2k}$ would be the real part of $(1+i)^{75}$. Hence converting to $1+i$ to polar form we have

$$(1+i)^{75}=(\sqrt{2},\pi/4)^{75}=\left(2^{75/2},\frac{75\pi}{4}\right)=\left(2^{75/2},\frac{3\pi}{4}\right)$$

Computing the real part we have $S=Re\left(2^{75/2},\frac{3\pi}{4}\right)=2^{75/2}\cos\frac{3\pi}{4}=\boxed{-2^{37}}$

8. Let $x_k=\omega^k$. Then

$$P=\prod_{k=0}^{6}(1+x_k-x_k^2)=-\prod_{k=0}^{6}(\phi-x_k)(\tau-x_k)$$

where ϕ,τ are the two solutions to x^2-x-1. Since ω is a primitive root of unity, as we go over all powers we pick up all the roots of unity. Hence we must have that $P=-(\phi-1)(\phi-\omega)(\phi-\omega^2)\ldots(\phi-\omega^6)(\tau-1)(\tau-\omega)(\tau-\omega^2)\ldots(\tau-\omega^6)=-(\phi^7-1)(\tau^7-1)$. Since both ϕ,τ satisfy $x^2=x+1$, we have that

$$\begin{aligned}x^7-1=x*x^6-1&=x(x+1)^3-1\\ &=x(x^3+3x^2+3x+1)-1\\ &=x^2(x^2+3x+3)+x-1\\ &=(x+1)(4x+4)+x-1\\ &=4x^2+9x+3\\ &=13x+7\end{aligned}$$

Now since $\phi + \tau = 1, \phi\tau = -1$, we have that

$$P = -(13\phi + 7)(13\tau + 7) = -(-169 + 91 + 49) = \boxed{29}$$

9. [1] We wish to find

$$L = \lim_{n\to\infty} \frac{1}{n^2}\left(\sqrt{n^2-1} + \sqrt{n^2-2^2} + \ldots + \sqrt{n^2-(n-1)^2}\right)$$

Bringing a factor of $1/n$ into the square roots we have

$$\begin{aligned} L &= \lim_{n\to\infty} \frac{1}{n}\left(\sqrt{1-\frac{1}{n^2}} + \sqrt{1-\frac{2^2}{n^2}} + \ldots + \sqrt{1-\frac{(n-1)^2}{n^2}}\right) \\ &= \lim_{n\to\infty} \frac{1}{n}\sum_{k=1}^{n}\sqrt{1-\left(\frac{k}{n}\right)^2} \end{aligned}$$

Notice this is a Riemann sum with $\Delta = \frac{1}{n}$ and $x_k^* = \frac{k}{n}$. In the limit as n goes to infinity this converges to the integral $\int_0^1 \sqrt{1-x^2}\ \mathrm{d}x$. This is just one quarter of the area of the unit circle so the final answer is $\boxed{\frac{\pi}{4}}$

10. We have that $I = \int_0^{\pi/2} \ln(4\sin x)\ \mathrm{d}x = \int_0^{\pi/2} \ln(4\cos x)\ \mathrm{d}x$ since $\sin x$ and $\cos x$ take on the same values on the interval $[0, \pi/2]$. Adding these we have

$$\begin{aligned} 2I &= \int_0^{\pi/2} \ln(4\sin x)\ \mathrm{d}x + \int_0^{\pi/2} \ln(4\cos x)\ \mathrm{d}x \\ &= \int_0^{\pi/2} \ln(16\sin x\cos x)\ \mathrm{d}x \\ &= \int_0^{\pi/2} \ln(16\sin x\cos x)\ \mathrm{d}x \\ &= \int_0^{\pi/2} \ln(2) + \ln(8\sin x\cos x)\ \mathrm{d}x \\ &= \frac{\pi\ln 2}{2} + \int_0^{\pi/2} \ln(4\sin 2x)\ \mathrm{d}x \end{aligned}$$

Making the subsitution $u = 2x$ and noting that $\sin x$ assumes the same values from $[\pi/2, \pi]$ as $[0, \pi/2]$, we have:

$$\begin{aligned} 2I &= \frac{\pi\ln 2}{2} + \frac{1}{2}\int_0^{\pi} \ln(4\sin u)\ \mathrm{d}u \\ &= \frac{\pi\ln 2}{2} + \frac{1}{2}\left(\int_0^{\pi/2} \ln(4\sin u)\ \mathrm{d}u + \int_{\pi/2}^{\pi} \ln(4\sin u)\ \mathrm{d}u\right) \\ &= \frac{\pi\ln 2}{2} + \frac{2I}{2} \end{aligned}$$

Hence we have $I = \boxed{\frac{\pi\ln 2}{2}}$

[1] There was a typo in the actual test, $1/n^3$ should have been $1/n^2$