1. What is the value of $1+7+21+35+35+21+7+1$?

2. A mathematician is walking through a library with twenty-six shelves, one for each letter of the alphabet. As he walks, the mathematician will take at most one book off each shelf. He likes symmetry, so if the letter of a shelf has at least one *line* of symmetry (e.g., M works, L does not), he will pick a book with probability $\frac{1}{2}$. Otherwise he has a $\frac{1}{4}$ probability of taking a book. What is the expected number of books that the mathematician will take?

3. Together, Abe and Bob have less than or equal to \$100. When Corey asks them how much money they have, Abe says that the reciprocal of his money added to Bob's money is thirteen times as much as the sum of Abe's money and the reciprocal of Bob's money. If Abe and Bob both have integer amounts of money, how many possible values are there for Abe's money?

4. In a right triangle, the altitude from a vertex to the hypotenuse splits the hypotenuse into two segments of lengths a and b. If the right triangle has area T and is inscribed in a circle of area C, find ab in terms of T and C.

5. Call two regular polygons supplementary if the sum of an internal angle from each polygon adds up to 180. For instance, two squares are supplementary because the sum of the internal angles is $90+90=180$. Find the other pair of supplementary polygons. Write your answer in the form (m, n) where m and n are the number of sides of the polygons and $m<n$.

6. A train is going up a hill with vertical velocity given as a function of t by $\frac{1}{1-t^4}$, where t is between $[0,1)$. Determine its height as a function of t.

7. Let $VWXYZ$ be a square pyramid with vertex V with height 1, and with the unit square as its base. Let $STANFURD$ be a cube, such that face $FURD$ lies in the same plane as and shares the same center as square face $WXYZ$. Furthermore, all sides of $FURD$ are parallel to the sides of $WXYZ$. Cube $STANFURD$ has side length s such that the volume that lies inside the cube but outside the square pyramid is equal to the volume that lies inside the square pyramid but outside the cube. What is the value of s?

8. Annisa has n distinct textbooks, where $n>6$. She has a different ways to pick a group of 4 books, b different ways to pick 5 books and c different ways to pick 6 books. If Annisa buys two more (distinct) textbooks, how many ways will she be able to pick a group of 6 books?

9. Two different functions f, g of x are selected from the set of real-valued functions

$$\left\{\sin x,\ e^{-x},\ x\ln x,\ \arctan x,\ \sqrt{x^2+x}-\sqrt{x^2-x},\ \frac{1}{x}\right\}$$

to create a product function $f(x)g(x)$. For how many such products is $\lim_{x\to\infty} f(x)g(x)$ finite?

10. A unitary divisor d of a number n is a divisor n that has the property $\gcd(d, n/d)=1$. If $n=1620$, what is the sum of all of the unitary divisors of d?

11. Suppose that $x^{10}+x+1=0$ and $x^{100}=a_0+a_1x+\ldots+a_9x^9$. Find a_5.

12. A two-digit integer is reversible if, when written backwards in base 10, it has the same number of positive divisors. Find the number of reversible integers.

13. Let ABC be a triangle with $AB = 16, AC = 10, BC = 18$. Let D be a point on AB such that $4AD = AB$ and let E be the foot of the angle bisector from B onto AC. Let P be the intersection of CD and BE. Find the area of the quadrilateral $ADPE$.

14. Let (x, y) be an intersection of the equations $y = 4x^2 - 28x + 41$ and $x^2 + 25y^2 - 7x + 100y + \frac{349}{4} = 0$. Find the sum of all possible values of x.

15. Suppose a box contains 28 balls: 1 red, 2 blue, 3 yellow, 4 orange, 5 purple, 6 green, and 7 pink. One by one, each ball is removed uniformly at random and without replacement until all 28 balls have been removed. Determine the probability that the most likely "scenario of exhaustion" occurs; that is, determine the probability that the first color to have all such balls removed from the box is red, that the second is blue, the third is yellow, the fourth is orange, the fifth is purple, the sixth is green, and the seventh is pink.

16. Evaluate
$$\sum_{n=0}^{\infty}\sum_{k=0}^{\infty}\min(n,k)\left(\frac{1}{2}\right)^n\left(\frac{1}{3}\right)^k.$$

17. Suppose you started at the origin on the number line in a coin-flipping game. Every time you flip a heads, you move forward one step, otherwise you move back one step. However, there are walls at positions 8 and -8; if you are at these positions and your coin flip dictates that you should move past them, instead you must stay. What is the expected number of coin flips needed to have visited both walls?

18. Monty wants to play a game with you. He shows you five boxes, one of which contains a prize and four of which contain nothing. He allows you to choose one box but not to open it. He then opens one of the other four boxes that he knows to contain nothing. Then, he makes you switch and choose a different, unopened box. However, Monty sketchily reveals the contents of another (empty) box, selected uniformly at random from the two or three closed boxes (that you do not currently have chosen) that he knows to contain no prize. He then offers you the chance to switch again. Assuming you seek to maximize your return, determine the probability you get a prize.

19. A number k is *nice* in base b if there exists a k-digit number n such that $n,\ 2n, \ldots kn$ are each some cyclic shifts of the digits of n in base b (for example, 2 is *nice* in base 5 because $2 \cdot 13_5 = 31_5$). Determine all nice numbers in base 18.

20. A certain type of Bessel function has the form $I(x) = \frac{1}{\pi}\int_0^{\pi} e^{x\cos\theta}\,d\theta$ for all real x. Evaluate
$$\int_0^{\infty} xI(2x)e^{-x^2}\,dx.$$