1. Find all real numbers x such that $4^x - 2^{x+2} + 3 = 0$.

2. Find the smallest positive value of x such that $x^3 - 9x^2 + 22x - 16 = 0$.

3. Emma is seated on a train traveling at a speed of 120 miles per hour. She notices distance markers are placed evenly alongside the track, with a constant distance x between any two consecutive ones, and during a span of 6 minutes, she sees precisely 11 markers pass by her. Determine the difference (in miles) between the largest and smallest possible values of x.

4. The function $f(x) = x^5 - 20x^4 + ax^3 + bx^2 + cx + 24$ has the interesting property that its roots can be arranged to form an arithmetic sequence. Determine $f(8)$.

5. Determine
$$\lim_{x\to\infty} \frac{\sqrt{x+2014}}{\sqrt{x} + \sqrt{x+2014}}.$$

6. Find $f(2)$ given that f is a real-valued function that satisfies the equation
$$4f(x) + \left(\frac{2}{3}\right)(x^2+2)f\left(x - \frac{2}{x}\right) = x^3 + 1.$$

7. Let $f(x) = x^2 + 18$ have roots r_1 and r_2, and let $g(x) = x^2 - 8x + 17$ have roots r_3 and r_4. If $h(x) = x^4 + ax^3 + bx^2 + cx + d$ has roots $r_1 + r_3$, $r_1 + r_4$, $r_2 + r_3$, and $r_2 + r_4$, then find $h(4)$.

8. Suppose an integer-valued function f satisfies
$$\sum_{k=0}^{2n+1} f(k) = \ln|2n+1| - 4\ln|2n-1| \text{ and } \sum_{k=0}^{2n} f(k) = 4e^n - e^{n-1}$$
for all non-negative integers n. Determine $\displaystyle\sum_{n=0}^{\infty} \frac{f(n)}{2^n}$.

9. Find α such that
$$\lim_{x\to 0^+} x^\alpha I(x) = a \quad \text{given} \quad I(x) = \int_0^\infty \sqrt{1+t}\cdot e^{-xt}\,dt$$
where a is a nonzero real number.

10. Suppose that $x^3 - x + 10^{-6} = 0$. Suppose that $x_1 < x_2 < x_3$ are the solutions for x. Find the integers (a, b, c) closest to $10^8 x_1$, $10^8 x_2$, and $10^8 x_3$ respectively.

P1. Suppose that a, b, c, d are non-negative real numbers such that $a^2 + b^2 + c^2 + d^2 = 2$ and $ab + bc + cd + da = 1$. Find the maximum value of $a + b + c + d$ and determine all equality cases.

P2. Define $\eta(f)$ to be the number of roots that are repeated of the complex-valued polynomial f (e.g., $\eta((x-1)^3 \cdot (x+1)^4) = 5$). Prove that for nonconstant, relatively prime $f, g \in \mathbb{C}[x]$,
$$\eta(f) + \eta(g) + \eta(f+g) < \deg f + \deg g.$$