1. A rectangle with sides a and b has an area of 24 and a diagonal of length 11. Find the perimeter of this rectangle.

2. Two rays start from a common point and have an angle of 60 degrees. Circle C is drawn with radius 42 such that it is tangent to the two rays. Find the radius of the circle that has radius smaller than circle C and is also tangent to C and the two rays.

3. Given a regular tetrahedron $ABCD$ with center O, find $\sin\angle AOB$.

4. Two cubes A and B have different side lengths, such that the volume of cube A is numerically equal to the surface area of cube B. If the surface area of cube A is numerically equal to six times the side length of cube B, what is the ratio of the surface area of cube A to the volume of cube B?

5. Points A and B are fixed points in the plane such that $AB = 1$. Find the area of the region consisting of all points P such that $\angle APB > 120°$.

6. Let $ABCD$ be a cyclic quadrilateral where $AB = 4$, $BC = 11$, $CD = 8$, and $DA = 5$. If BC and DA intersect at X, find the area of $\triangle XAB$.

7. Let ABC be a triangle with $BC = 5$, $CA = 3$, and $AB = 4$. Variable points P, Q are on segments AB, AC, respectively such that the area of APQ is half of the area of ABC. Let x and y be the lengths of perpendiculars drawn from the midpoint of PQ to sides AB and AC, respectively. Find the range of values of $2y + 3x$.

8. ABC is an isosceles right triangle with right angle B and $AB = 1$. ABC has an incenter at E. The excircle to ABC at side AC is drawn and has center P. Let this excircle be tangent to AB at R. Draw T on the excircle so that RT is the diameter. Extend line BC and draw point D on BC so that DT is perpendicular to RT. Extend AC and let it intersect with DT at G. Let F be the incenter of CDG. Find the area of $\triangle EFP$.

9. Let ABC be a triangle. Points D, E, F are on segments BC, CA, AB, respectively. Suppoe that $AF = 10$, $FB = 10$, $BD = 12$, $DC = 17$, $CE = 11$, and $EA = 10$. Suppose that the circumcircles of $\triangle BFD$ and $\triangle CED$ intersect again at X. Find the circumradius of $\triangle EXF$.

10. Let D, E, and F be the points at which the incircle, ω, of $\triangle ABC$ is tangent to BC, CA, and AB, respectively. AD intersects ω again at T. Extend rays TE, TF to hit line BC at E', F', respectively. If $BC = 21$, $CA = 16$, and $AB = 15$, then find $\left|\dfrac{1}{DE'} - \dfrac{1}{DF'}\right|$.

P1. Suppose a convex polygon has a perimeter of 1. Prove that it can be covered with a circle of radius 1/4.

P2. From a point A construct tangents to a circle centered at point O, intersecting the circle at P and Q respectively. Let M be the midpoint of PQ. If K and L are points on circle O such that K, L, and A are collinear, prove $\angle MKO = \angle MLO$.