INDIVIDUAL ROUND

1. Let $\{a_n\}_{n=1}^{\infty}$ be an arithmetic progression with $a_1 > 0$ and $5a_{13} = 6a_{19}$. What is the smallest integer n such that $a_n < 0$?

2. Find the smallest number with exactly 28 divisors.

3. Find the largest prime factor of
$$\frac{1}{\sum_{n=1}^{\infty} \frac{2012}{n(n+1)(n+2)\cdots(n+2012)}}$$

4. Tyler rolls two 4025-sided fair dice with sides numbered $1, \ldots, 4025$. Given that the number on the first die is greater than or equal to the number on the second die, what is the probability that the number on the first die is less than or equal to 2012?

5. Let $a \uparrow\uparrow b = \overbrace{a^{a^{a^{\cdot^{\cdot^{\cdot^{a}}}}}}}^{b}$. That is, $a \uparrow\uparrow b$ is given by the recurrence
$$a \uparrow\uparrow b = \begin{cases} a & b = 1 \\ a^{a \uparrow\uparrow (b-1)} & b \geq 2 \end{cases}$$
What is the remainder of $3 \uparrow\uparrow (3 \uparrow\uparrow (3 \uparrow\uparrow 3))$ when divided by 60?

6. Let $ABCD$ be a cyclic quadrilateral, with $AB = 7$, $BC = 11$, $CD = 13$, and $DA = 17$. Let the incircle of ABD hit BD at R and the incircle of CBD hit BD at S. what is RS?

7. Let a, b, c, d, $(a+b+c+18+d)$, $(a+b+c+18-d)$, $(b+c)$, and $(c+d)$ be distinct prime numbers such that $a+b+c = 2010$, $a, b, c, d \neq 3$, and $d \leq 50$. Find the maximum value of the difference between two of these prime numbers.

8. You are tossing an unbiased coin. The last 28 consecutive flips have all resulted in heads. Let x be the expected number of additional tosses you must make before you get 60 consecutive heads. Find the sum of all distinct prime factors in x.

9. Bowling Pins is a game played between two players in the following way:

   We start with 14 bowling pins in a line:

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X X X X X X X X X X X X X X
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   Players alternate turns. On each turn, the player can knock down one, two or three consecutive pins at a time. For example:

   Jing Jing bowls:

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X X     X X X X X X X X X X
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   Soumya bowls:

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X X     X X X   X X X X X X
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   Jing Jing bowls again:

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X X     X X X   X X       X
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   The player who knocks down the last pin wins.

   In the above game, it is Soumya's turn. If he plays perfectly from here, he has a winning strategy (In fact, he has four different winning moves.)

   Imagine it's Jing Jing's turn to play and the game looks as follows

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X       X...X
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   with 1 X on the left and a string of k consecutive X's on the right.

   For what values of k from 1 to 10 does she have a winning strategy?

10. You are at one vertex of a equilateral triangle with side length 1. All of the edges of the equilateral triangle will reflect the laser beam perfectly (angle of incidence is equal to angle of reflection). Given that the laser beam bounces off exactly 137 edges and returns to the original vertex without touching any other vertices, let M be the maximum possible distance the beam could have traveled, and m be the minimum possible distance the beam could have traveled. Find $M^2 - m^2$.