**Time limit:** 30 minutes.
**Instructions:** For this test, you work in teams of five to solve 20 short answer questions. All answers must be expressed in simplest form unless specified otherwise. Submit a single answer sheet for grading. Only answers written inside the boxes on the answer sheet will be considered for grading.
**No calculators.**

1. What is the sum of the first 12 positive integers?

2. How many positive integers less than or equal to 100 are multiples of both 2 and 5?

3. Alex has a bag with 4 white marbles and 4 black marbles. She takes 2 marbles from the bag without replacement. What is the probability that both marbles she took are black? Express your answer as a decimal or a fraction in lowest terms.

4. How many 5-digit numbers are there where each digit is either 1 or 2?

5. An integer $a$ with $1 \le a \le 10$ is randomly selected. What is the probability that $\frac{100}{a}$ is an integer? Express your answer as decimal or a fraction in lowest terms.

6. Two distinct non-tangent circles are drawn so that they intersect each other. A third circle, distinct from the previous two, is drawn. Let $P$ be the number of points of intersection between any two circles. How many possible values of $P$ are there?

7. Let $x, y, z$ be nonzero real numbers such that $x + y + z = xyz$. Compute
$$\frac{1 + yz}{yz} + \frac{1 + xz}{xz} + \frac{1 + xy}{xy}.$$

8. How many positive integers less than $10^6$ are simultaneously perfect squares, cubes, and fourth powers?

9. Let $C_1$ and $C_2$ be two circles centered at point $O$ of radii 1 and 2, respectively. Let $A$ be a point on $C_2$. We draw the two lines tangent to $C_1$ that pass through $A$, and label their other intersections with $C_2$ as $B$ and $C$. Let $x$ be the length of minor arc $BC$, as shown. Compute $x$.
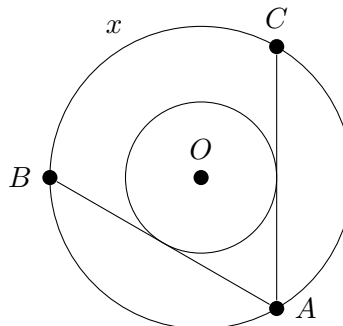


10. A circle of area $\pi$ is inscribed in an equilateral triangle. Find the area of the triangle.

11. Julie runs a 2 mile route every morning. She notices that if she jogs the route 2 miles per hour faster than normal, then she will finish the route 5 minutes faster. How fast (in miles per hour) does she normally jog?

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12. Let $ABCD$ be a square of side length 10. Let $EFGH$ be a square of side length 15 such that $E$ is the center of $ABCD$, $EF$ intersects $BC$ at $X$, and $EH$ intersects $CD$ at $Y$ (shown below). If $BX = 7$, what is the area of quadrilateral $EXCY$?
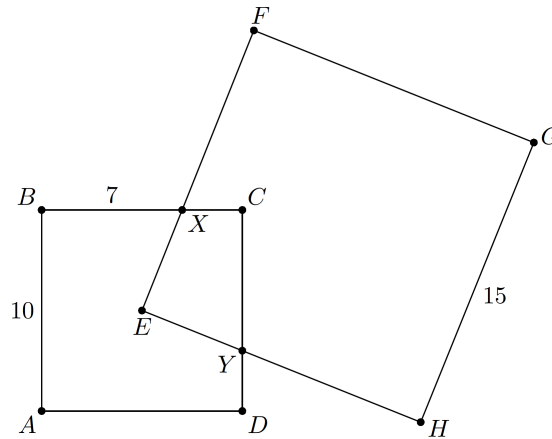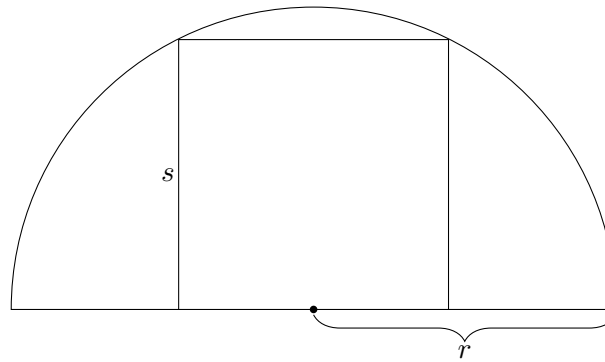


13. How many solutions are there to the system of equations

$$a^2 + b^2 = c^2$$
$$(a+1)^2 + (b+1)^2 = (c+1)^2$$

   if $a$, $b$, and $c$ are positive integers?

14. A square of side length $s$ is inscribed in a semicircle of radius $r$ as shown. Compute $\frac{s}{r}$.



15. $S$ is a collection of integers $n$ with $1 \leq n \leq 50$ so that each integer in $S$ is composite and relatively prime to every other integer in $S$. What is the largest possible number of integers in $S$?

16. Let $ABCD$ be a regular tetrahedron and let $W, X, Y, Z$ denote the centers of faces $ABC$, $BCD$, $CDA$, and $DAB$, respectively. What is the ratio of the volumes of tetrahedrons $WXYZ$ and $WAYZ$? Express your answer as a decimal or a fraction in lowest terms.

17. Consider a random permutation $\{s_1, s_2, \ldots, s_8\}$ of $\{1, 1, 1, 1, -1, -1, -1, -1\}$. Let $S$ be the largest of the numbers

$$s_1, \quad s_1 + s_2, \quad s_1 + s_2 + s_3, \quad \ldots, \quad s_1 + s_2 + \cdots + s_8.$$

   What is the probability that $S$ is exactly 3? Express your answer as a decimal or a fraction in lowest terms.

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18. A positive integer is called *almost-kinda-semi-prime* if it has a prime number of positive integer divisors. Given that there are 168 primes less than 1000, how many almost-kinda-semi-prime numbers are there less than 1000?

19. Let $ABCD$ be a unit square and let $X$, $Y$, $Z$ be points on sides $AB$, $BC$, $CD$, respectively, such that $AX = BY = CZ$. If the area of triangle $XYZ$ is $\frac{1}{3}$, what is the maximum value of the ratio $XB/AX$?
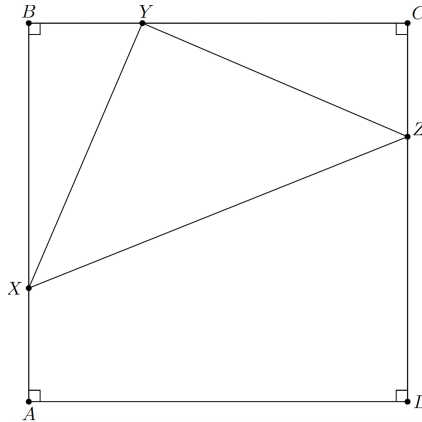


20. Positive integers $a \leq b \leq c$ have the property that each of $a + b$, $b + c$, and $c + a$ are prime. If $a + b + c$ has exactly 4 positive divisors, find the fourth smallest possible value of the product $c(c + b)(c + b + a)$.