

### Kagami

In Kagami, you will receive a grid, and you must fill in each square with either a positive integer or a diagonally-oriented mirror. Each row and column contains exactly one copy of the numbers from 1 to  $n$  for some positive integer  $n$ ; you must deduce what  $n$  is. Every square that does not contain a number has a diagonally-oriented mirror. For example, if we have a  $4 \times 4$  grid and  $n = 2$ , then you must fill each row and column with one 1, one 2, and two mirrors.

From each number outside of the grid, we draw a path straight into the grid. The path passes straight through numbers and bounces off of each mirror it hits at a right angle. The path then exits the grid at some other number. For a solution to be correct, the sum of the numbers that the path passes through must equal the numbers outside the grid that the path connects. If a path passes through a number twice, then the number is included twice in the sum. For example, below is a Kagami grid and its solution.

	3	0	0	6	
3					11
3					0
3					0
0					1
	0	6	1	11	

	3	0	0	6	
3	1	\	/	2	11
3	2	/	1	/	0
3	/	1	2	\	0
0	\	2	/	1	1
	0	6	1	11	

The grid below to the left shows this solution with two of the paths drawn. Note that the paths connect two equal numbers outside the grid, and that these numbers are equal to the sum of the numbers the path passes through. That's how we know this solution is correct!

	3	0	0	6	
3	1	\	/	2	11
3	2	/	1	/	0
3	/	1	2	\	0
0	\	2	/	1	1
	0	6	1	11	

	3	0	0	6	
3	1	\	/	2	11
3				/	0
3				\	0
0				1	1
	0	6	1	11	

The grid to the right above shows an example of how to start solving the puzzle. We first figured out that  $n = 2$ : if  $n > 2$ , then there wouldn't be enough mirrors to make sure there are 0's in the places shown, and if  $n < 2$ , then we wouldn't be able to make a path sum of 11. Because of the 0's at the top and right, we see that mirrors need to be placed as shown. The square at the bottom right must be a 1 or a 2, and it can't be 2 because of the 1 next to it. We now can fill in the other numbers shown using process of elimination.

Some quick reminders before you start:

- Every square must have either a number or a mirror.
- Paths are like lasers: they go straight until they hit a mirror, and then they bounce off the mirror at a right angle.
- You need to figure out what  $n$  is. In a  $m \times m$  grid,  $n$  is an integer between 0 and  $m$ .

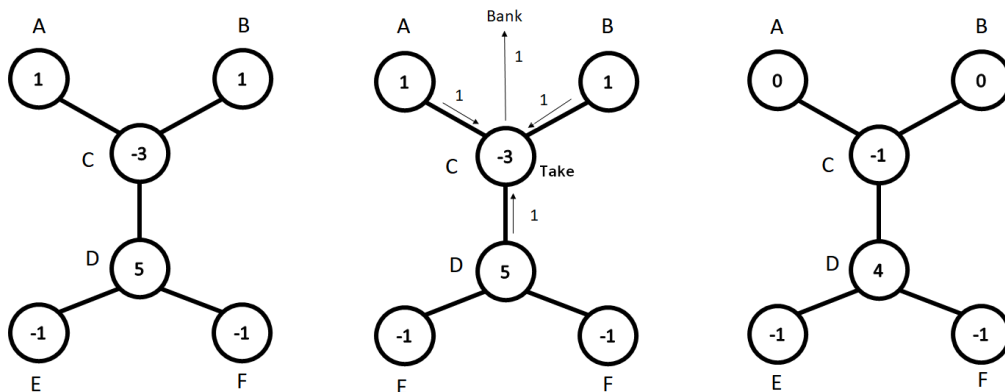
### Penny Pushers

In Penny Pushers, you will receive a collection of nodes, with edges connecting some of them. Each node represents a person, labelled with how much money that person has. Your goal is to make each person have \$0 by performing a sequence of moves. There are two valid moves: GIVE and TAKE.

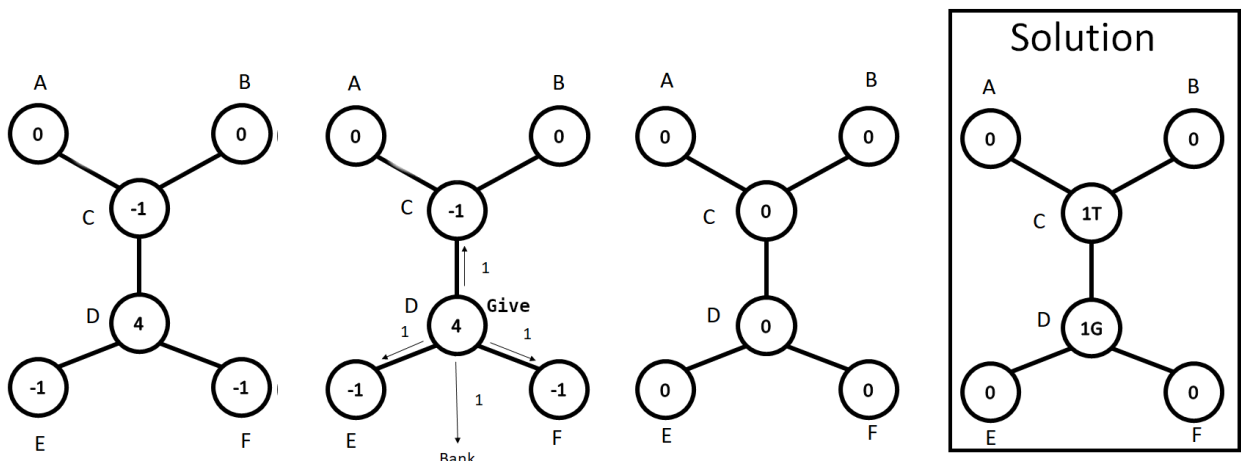
- When a node GIVES, it sends \$1 to each connected node, and it loses \$1 to the bank.
- When a node TAKES, it receives \$1 from each connected node, and it loses \$1 to the bank.

For example, suppose you are given the Penny Pushers problem shown below to the left (labels have been added to each node). Our first move is a node C TAKE, which is shown below. Since node C is connected to nodes A, B, and D, it takes \$1 from each of these nodes; node C additionally loses \$1 to the bank, so it overall gains  $\$3 - \$1 = \$2$ . Carefully examine the figure below to see the results of this node C TAKE.

#### Problem



The next move is a node D GIVE, which is shown below. Since node D is connected to nodes C, E, and F, it will send \$1 to each of these nodes; node D additionally loses \$1 to the bank, so it overall loses  $\$3 + \$1 = \$4$ . After this move, all nodes have \$0, so we have solved the puzzle!



To submit your answer, write the number of GIVES or TAKES each node performs, along with a G for GIVING or a T for TAKING. If a node did not perform any action, write a 0 with no letter. For example, in the problem above node C did one TAKE, node D did one GIVE, and all other nodes performed no actions. The correct way of submitting this answer is shown in the box above.

Some notes before you begin:

- The total amount of money decreases by \$1 each move. You can use this to figure out how many moves you need!
- The order in which you perform moves does *not* matter.
- A node with a negative balance can still GIVE.
- In the puzzles we give, no node will both GIVE and TAKE.
- Carefully examine the boxed figure above to make sure you understand how to submit your answer.

## Finale

The last section of the puzzle round consists of several extremely challenging puzzles. These questions are very tricky, and aren't worth that many points, so only try them if you've finished the rest of the questions.

:-)

Good luck and have fun!