Time limit: 10 minutes.
Instructions: For this test, you work in teams of five to solve 50 short answer questions. All answers must be expressed in simplest form unless specified otherwise. Submit a single answer sheet for grading. Only answers written inside the boxes on the answer sheet will be considered for grading.
No calculators.

1. True or False? 1 is prime.
2. True or False? For all real numbers a and b such that $a < b$, $a^2 < b^2$.
3. True or False? If x, y, and z are side lengths of triangle ABC, and $x^2 + y^2 > z^2$, then the triangle ABC is acute.
4. True or False? Let (a, b) and $[a, b]$ denote the greatest common divisor and least common multiple, respectively, of a and b. Then given any integers a, b, $\frac{[a,b]}{(a,b)}$ and (a, b) are relatively prime.
5. True or False? $x^3 = 3$ has 3 real solutions.
6. True or False? $12! > 12^{11}$.
7. True or False? If I draw some number of cards from a standard deck and don't replace them, it's more likely that I draw exactly 13 spades than that I draw exactly 14 spades.
8. True or False? There exist two points a and b on the boundary of a circle of radius r such that the distance from a to b is r.
9. True or False? Suppose p_1 and p_2 are planes that intersect. Then for any two lines ℓ_1 in p_1 and ℓ_2 in p_2, ℓ_1 and ℓ_2 must intersect.
10. True or False? 1729 can be written as the sum of two positive cubes in two different ways.
11. Is $2^{3^{4^5}}$ even or odd?
12. Is $(-1)^{100^2 \cdot 99} + 123 \cdot 456 \cdot 789 + 1 \cdot 11 \cdot 111 \cdot 1111$ even or odd?
13. Suppose that x is the largest number which solves the equation $0 = x^2 - 52x + 376$ and $[x]$ is the closest integer to x. Is $[x]$ even or odd?
14. Compute the sum of the digits of the first five prime numbers.
15. Compute the sum $1 - 2 + 3 - 4 + \cdots - 10$.
16. Compute 106^2.
17. Compute
$$1 - \frac{1}{2} + \frac{1}{4} - \frac{1}{8} + \frac{1}{16} - \frac{1}{32}.$$
18. Compute $\frac{1}{1\times 2} + \frac{1}{2\times 3} + \cdots + \frac{1}{16\times 17}$.
19. Compute the sum of the proper factors (that is, not 496 itself) of 496.

20. Compute $\sqrt{1^3 + 2^3 + 3^3 + \cdots + 10^3}$.

21. How many positive integers less than 50 have exactly 4 divisors?

22. How many ordered triples (a, b, c) are there such that $a \leq b \leq c$ and $abc = 1001$?

23. How many triangles with side length 1 tile a hexagon of side length 4?

24. There are 2 dogs for every cat at Nathan's pet shelter. If there are 36 dogs and cats, how many dogs are there?

25. Victoria can paint her house in 2 hours, and Clark can paint his house in 6 hours. If the area to be painted on Clark's house is twice as big as the area to be painted on Victoria's, how many hours would Victoria and Clark take to paint Victoria's house together?

26. How many positive integers n satisfy $n^{\frac{1}{n}} = \sqrt{2}$?

27. Let $ABCD$ be a square and let M be the midpoint of CD. How many points X are there on the boundary of $ABCD$ such that the area of AMX is one quarter of the area of $ABCD$?

28. How many ordered pairs (a, b) of integers are there with $a^2 + b^2 = 25$?

29. Three distinct circles are drawn on the plane. There are n points that lie on at least 2 circles. How many possible values are there for n?

30. What is the largest perfect square that divides $7^6 - 1$?

31. What is the leftmost digit of $\sqrt{99999}$?

32. Let (x, y) be an integer solution to
$$x^2 - 163y^2 = 1$$
such that $|x + y|$ is minimal. What is $|x + y|$?

33. Determine the units digit of $1^{10} + 10^9 + 110^8 + 1110^7$.

34. Find the area between the lines $y = 0$, $x = 2$, and $y = 3x$.

35. What is the slope of the line tangent to the curve $y = 1 - x^2$, when $x = 0$?

36. If I travel around the unit circle 16 times, how far have I traveled?

37. A square has a diagonal of length π. Calculate the area of the square, in terms of π.

38. Sanat, Ankit, and Moor are throwing a party. Each of them invites some number of guests, but because of poor planning, some guests may have been invited multiple times. Sanat invites 7 guests, Moor invites 12 guests and Ankit invites 6 guests. Compute the sum of all possible numbers of total invited guests.

39. If
$$\begin{aligned} x + y - z &= 42 \\ x - y + z &= 65 \\ -x + y + z &= 12, \end{aligned}$$
what is $x + y + z$?

40. Debbie is buying pineapples, durians, and candy. Pineapples cost 3 dollars apiece, durians 4 dollars apiece, and candies cost 1 dollar apiece. Debbie has 15 dollars to spend and she must spend all her money. What is the minimum number of items she can buy?

41. Some freshmen are building a zipline from their dorm and are testing the strength of the zipline with weights. Every time the rope holds, they add an additional weight to the zipline with a quarter of the weight of the previous weight that they added. They start with a 100 lb weight and the zipline breaks after they have added 200 weights. To the nearest integer, how much weight can the zipline support, in pounds?

42. The Jupiterian currency consists of \$1 and \$12 bills, and I only have \$12 bills. I'm at a restaurant ordering pot stickers that cost \$10 each. If I don't want any change, what's the minimum number of pot stickers I can order?

43. If n is an irreducible whole number such that $n+2$, n^2+2, n^3+2, n^4+2, and n^5+2 are all prime, what is n? (An irreducible whole number is a number n that is not reducible, that is it cannot be written as a product pq, where p and q are integers and neither p nor q are ± 1).

44. You stand on flat ground and throw an orange upwards. The orange travels along the trajectory of $y = 14t - 4.9t^2$, where t is the time in seconds after you release the orange. What is the orange's maximum height?

45. Cailan reserves rooms for BmMT problem writing sessions, which takes place from 6:30 to 8:30, but there's a $\frac{2}{3}$ chance that he makes a mistake and reserves the room until 7:30. If he makes a mistake, there's a $\frac{1}{2}$ chance that the problem writers will be allowed to stay until 8:30 anyways. What is the probability that the problem writers can stay in the reserved room until 8:30 for the next two meetings?

46. Let a_i be a sequence defined as follows:

    (i) $a_1 = 5$.

    (ii) a_{i+1} is the front digit of the value $\overline{a_1a_2a_3\ldots a_i}^2$, where $\overline{abc}$ is the concatenation of digits a, b, c. For example, if $a = 1$, $b = 2$, $c = 3$, then $\overline{abc}$ is 123.

    What is the value of $a_1 + a_2 + a_3 + a_4 + \cdots + a_{2017}$?

47. Define a recursive function $F_{n+1} = (F_n)^2 F_{n-1} + F_{n-1}$. If $F_0 = F_1 = 0$, determine F_{2017}.

48. Let x be the sum of positive integers less than or equal to 2017 that do not share any common factors greater than 1 with 2017. Compute $\frac{x}{2017}$. Express your answer as an integer or common fraction in lowest terms.

49. 2017 helicopters consume 2017 gallons of fuel in total over a span of 2017 seconds in UC Berkeley's air show. How much fuel does one helicopter consume over a span of a minute? Express your answer as a common fraction in lowest terms.

50. Alice has 2017 cards, labeled 1 through 2017. At first, the cards are stacked in order, with the 1 card on top. Alice chooses a random prime number p less than or equal to the number of cards in the deck, and removes the pth card from the top. Alice repeats this process 2016 times. What is the number on the card remaining?