Time limit: 60 minutes.
Instructions: This test contains 20 short answer questions. All answers must be expressed in simplest form unless specified otherwise. Only answers written inside the boxes on the answer sheet will be considered for grading.
No calculators.

1. It's currently 6:00 on a 12 hour clock. What time will be shown on the clock 100 hours from now? Express your answer in the form $hh:mm$.

2. A tub originally contains 10 gallons of water. Alex adds some water, increasing the amount of water by 20%. Barbara, unhappy with Alex's decision, decides to remove 20% of the water currently in the tub. How much water, in gallons, is left in the tub? Express your answer as an exact decimal.

3. There are 2000 math students and 4000 CS students at Berkeley. If 5580 students are either math students or CS students, then how many of them are studying both math and CS?

4. Determine the smallest integer x greater than 1 such that x^2 is one more than a multiple of 7.

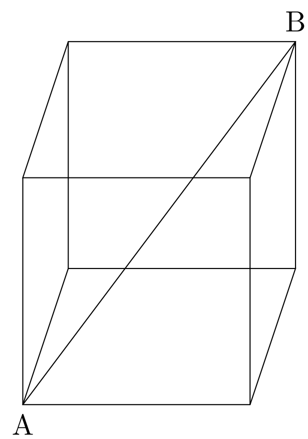
5. Find two positive integers x, y greater than 1 whose product equals the following sum:

$$9 + 11 + 13 + 15 + 17 + 19 + 21 + 23 + 25 + 27 + 29.$$

   Express your answer as an ordered pair (x, y) with $x \le y$.

6. The average walking speed of a cow is 5 meters per hour. If it takes the cow an entire day to walk around the edges of a perfect square, then determine the area (in square meters) of this square.

7. Consider the cube below. If the length of the diagonal AB is $3\sqrt{3}$, determine the volume of the cube.



8. I have 18 socks in my drawer, 6 colored red, 8 colored blue and 4 colored green. If I close my eyes and grab a bunch of socks, how many socks must I grab to guarantee there will be two pairs of matching socks?

9. Define the operation $a@b$ to be $3 + ab + a + 2b$. There exists a number x such that $x@b = 1$ for all b. Find x.

10. Compute the units digit of $2017^{(2017^2)}$.

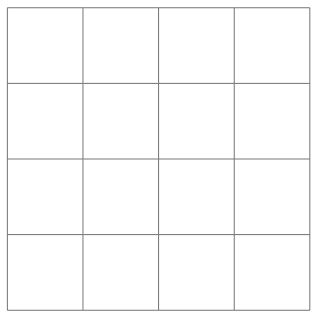
11. The distinct rational numbers $-\sqrt{-x}$, x, and $-x$ form an arithmetic sequence in that order. Determine the value of x.

12. Let $y = x^2 + bx + c$ be a quadratic function that has only one root. If b is positive, find $\frac{b+2}{\sqrt{c}+1}$.

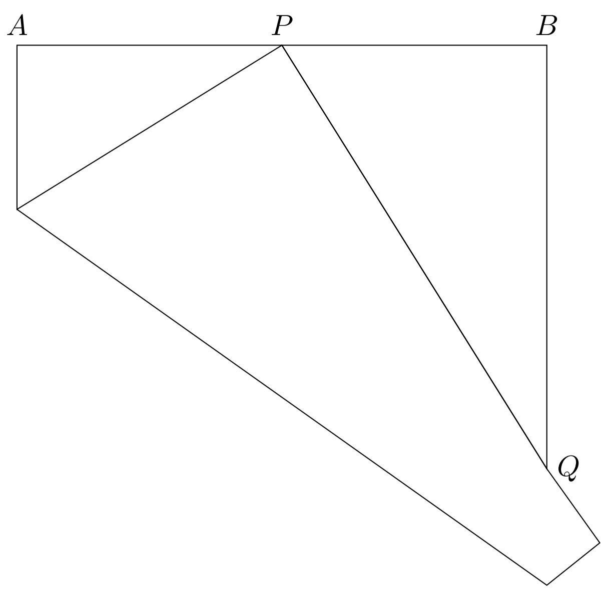
13. Alice, Bob, and four other people sit themselves around a circular table. What is the probability that Alice does not sit to the left or right of Bob?

14. Let $f(x) = |x - 8|$. Let p be the sum of all the values of x such that $f(f(f(x))) = 2$ and q be the minimum solution to $f(f(f(x))) = 2$. Compute $p \cdot q$.

15. Determine the total number of rectangles (1×1, 1×2, 2×2, etc.) formed by the lines in the figure below:



16. Take a square $ABCD$ of side length 1, and let P be the midpoint of AB. Fold the square so that point D touches P, and let the intersection of the bottom edge DC with the right edge be Q. What is BQ?



17. Let A, B, and k be integers, where k is positive and the greatest common divisor of A, B, and k is 1. Define $x\#y$ by the formula $x\#y = \frac{Ax+By}{kxy}$. If $8\#4 = \frac{1}{2}$ and $3\#1 = \frac{13}{6}$, determine the sum $A + B + k$.

18. There are 20 indistinguishable balls to be placed into bins A, B, C, D, and E. Each bin must have at least 2 balls inside of it. How many ways can the balls be placed into the bins, if each ball must be placed in a bin?

19. Let T_i be a sequence of equilateral triangles such that

    (a) T_1 is an equilateral triangle with side length 1.

    (b) T_{i+1} is inscribed in the circle inscribed in triangle T_i for $i \geq 1$.

    Find
$$\sum_{i=1}^{\infty} \text{Area}(T_i).$$

20. A *gorgeous* sequence is a sequence of 1's and 0's such that there are no consecutive 1's. For instance, the set of all *gorgeous* sequences of length 3 is $\{[1,0,0],[1,0,1],[0,1,0],[0,0,1],[0,0,0]\}$. Determine the number of *gorgeous* sequences of length 7.

Time limit: 15 minutes.
Instructions: This tiebreaker contains 5 short answer questions. All answers must be expressed in simplest form unless specified otherwise. You will submit answers to the problem as you solve them, and may solve problems in any order. You will not be informed whether your answer is correct until the end of the tiebreaker. You may submit multiple times for any of the problems, but **only the last submission for a given problem will be graded**. The participant who correctly answers the most problems wins the tiebreaker, with ties broken by the time of the last correct submission.
No calculators.

1. Consider a 4×4 lattice on the coordinate plane. At (0,0) is Mori's house, and at (4,4) is Mori's workplace. Every morning, Mori goes to work by choosing a path going up and right along the roads on the lattice. Recently, the intersection at $(2, 2)$ was closed. How many ways are there now for Mori to go to work?

2. Given two integers, define an operation $*$ such that if a and b are integers, then $a * b$ is an integer. The operation $*$ has the following properties:

   1. $a * a = 0$ for all integers a.
   2. $(ka + b) * a = b * a$ for integers a, b, k.
   3. $0 \leq b * a < a$.
   4. If $0 \leq b < a$, then $b * a = b$.

   Find $2017 * 16$.

3. Let ABC be a triangle with side lengths $AB = 13$, $BC = 14$, $CA = 15$. Let A', B', C', be the midpoints of BC, CA, and AB, respectively. What is the ratio of the area of triangle ABC to the area of triangle $A'B'C'$?

4. In a strange world, each orange has a label, a number from 0 to 10 inclusive, and there are an infinite number of oranges of each label. Oranges with the same label are considered indistinguishable. Sally has 3 boxes, and randomly puts oranges in her boxes such that

   (a) If she puts an orange labelled a in a box (where a is any number from 0 to 10), she cannot put any other oranges labelled a in that box.
   (b) If any two boxes contain an orange that have the same labelling, the third box must also contain an orange with that labelling.
   (c) The three boxes collectively contain all types of oranges (oranges of any label).

   The number of possible ways Sally can put oranges in her 3 boxes is N, which can be written as the product of primes:
   $$p_1^{e_1} p_2^{e_2} \cdots p_k^{e_k}$$
   where $p_1 \neq p_2 \neq p_3 \cdots \neq p_k$ and p_i are all primes and e_i are all positive integers. What is the sum $e_1 + e_2 + e_3 + \cdots + e_k$?

5. Suppose I want to stack 2017 identical boxes. After placing the first box, every subsequent box must either be placed on top of another one or begin a new stack to the right of the rightmost pile. How many different ways can I stack the boxes, if the order I stack them doesn't matter? Express your answer as
   $$p_1^{e_1} p_2^{e_2} \cdots p_n^{e_n}$$
   where $p_1, p_2, p_3, \ldots, p_n$ are distinct primes and e_i are all positive integers.