Time limit: 30 minutes.
Instructions: For this test, you work in teams of five to solve 20 short answer questions. All answers must be expressed in simplest form unless specified otherwise. Submit a single answer sheet for grading. Only answers written inside the boxes on the answer sheet will be considered for grading.
No calculators.

1. BmMT is in a week, and we don't have any problems! Let's write 1 on the first day, 2 on the second day, 4 on the third, 8 on the fourth, 16 on the fifth, 32 on the sixth, and 64 on the seventh. After seven days, how many problems will we have written in total?

2. 100 students are taking a ten-point exam. 50 students scored 8 points, 30 students scored 7 points, and the rest scored 9 points. What is the average score for the exam?

3. Rebecca has four pairs of shoes. Rebecca may or may not wear matching shoes. However, she will always use a left-shoe for her left foot and a right-shoe for her right foot. How many ways can Rebecca wear shoes?

4. A council of 111 mathematicians voted on whether to hold their conference in Beijing or Shanghai. The outcome of an initial vote was 70 votes in favor of Beijing, and 41 votes in favor of Shanghai. If the vote were to be held again, what is the minimum number of mathematicians that would have to change their votes in order for Shanghai to win a majority of votes?

5. What is the area of the triangle bounded by the line $20x + 16y = 160$, the x-axis, and the y-axis?

6. Suppose that 3 runners start running from the start line around a circular 800-meter track and that their speeds are 100, 160, and 200 meters per minute, respectively. How many minutes will they run before all three are next at the start line at the same time?

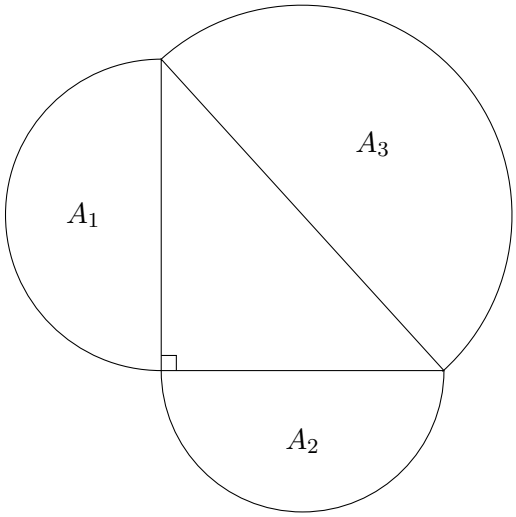
7. Brian's lawn is in the shape of a circle, with radius 10 meters. Brian can throw a frisbee up to 50 meters from where he stands. What is the area of the region (in square meters) in which the frisbee can land, if Brian can stand anywhere on his lawn?

8. A seven digit number is called "bad" if exactly four of its digits are 0 and the rest are odd. How many seven digit numbers are bad?

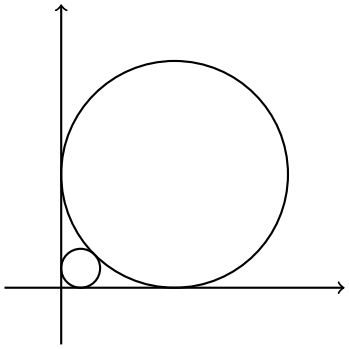
9. Suppose you have a 3-digit number with only even digits. What is the probability that twice that number also has only even digits?

10. You have a flight on Air China from Beijing to New York. The flight will depart any time between 1 p.m. and 6 p.m., uniformly at random. Your friend, Henry, is flying American Airlines, also from Beijing to New York. Henry's flight will depart any time between 3 p.m. and 5 p.m., uniformly at random. What is the probability that Henry's flight departs before your flight?

11. In the figure below, three semicircles are drawn outside the given right triangle. Given the areas $A_1 = 17$ and $A_2 = 14$, find the area A_3.



12. Consider a circle of radius 1 drawn tangent to the positive x and y axes. Now consider another smaller circle tangent to that circle and also tangent to the positive x and y axes. Find the radius of the smaller circle.



13. The following expression is an integer. Find this integer:

$$\frac{\sqrt{20+16\frac{\sqrt{20+16\frac{\sqrt{20+16\ldots}}{2}}}{2}}}{2}.$$

14. Let $2016 = a_1 \times a_2 \times \cdots \times a_n$ for some positive integers $a_1, a_2, \ldots, a_n$. Compute the smallest possible value of $a_1 + a_2 + \cdots + a_n$.

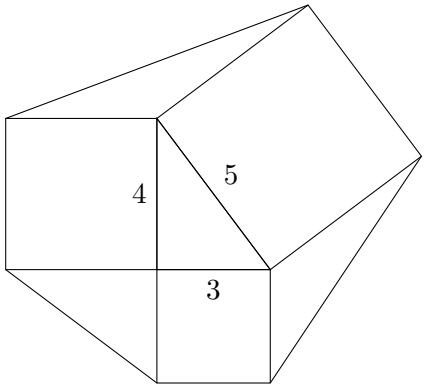
15. The tetranacci numbers are defined by the recurrence $T_n = T_{n-1} + T_{n-2} + T_{n-3} + T_{n-4}$ and $T_0 = T_1 = T_2 = 0$ and $T_3 = 1$. Given that $T_9 = 29$ and $T_{14} = 773$, calculate T_{15}.

16. Find the number of zeros at the end of $(2016!)^{2016}$. Your answer should be an integer, not its prime factorization.

17. A DJ has 7 songs named 1, 2, 3, 4, 5, 6, and 7. He decides that no two even-numbered songs can be played one after the other. In how many different orders can the DJ play the 7 songs?

18. Given a cube, how many distinct ways are there (using 6 colors) to color each face a distinct color? Colorings are distinct if they cannot be transformed into one another by a sequence of rotations.

19. Suppose you have a triangle with side lengths 3, 4, and 5. For each of the triangle's sides, draw a square on its outside. Connect the adjacent vertices in order, forming 3 new triangles (as in the diagram). What is the area of this convex region?



20. Find x such that $\sqrt{c+\sqrt{c-x}} = x$ when $c = 4$.