

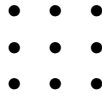
**Time limit:** 10 minutes.

**Instructions:** For this test, you work in teams of five to solve 50 short answer questions. All answers must be expressed in simplest form unless specified otherwise. Submit a single answer sheet for grading. Only answers written inside the boxes on the answer sheet will be considered for grading.

**No calculators.**

1. True or False? Your answer to this question will be True.
2. True or False? If two polygons are equiangular, then they are similar.
3. True or False?  $5^9 > 9!$ .
4. True or False? If a positive integer  $x$  has exactly 3 distinct divisors, it must be a perfect square.
5. True or False? If  $x$  is a perfect square, it must have exactly 3 distinct divisors.
6. True or False? Let  $f(x, y) = 4x^2 + 12xy + 9y^2 + 1$ . There exist real numbers  $a$  and  $b$  such that  $f(a, b) = 0$ .
7. True or False? If you buy two 12-inch diameter pizzas, you get more pizza than if you buy one 17-inch diameter pizza.
8. True or False? When flipping an unbiased coin 10 times, the sequence  $HTHHTHTHTH$  is more likely to occur than the sequence  $HHHHHHHHHH$ .
9. True or False? Out of the integers in the range from 1 to 16 inclusive, more numbers are prime than are perfect squares or perfect cubes.
10. True or False? The probability of rolling 2 dice and getting a sum of at least 7 is greater than the probability of flipping 6 coins and getting at least 3 heads.
11. Compute  $1 + 1$ .
12. Compute  $29 \times 31$ .
13. Compute  $1 + 3 - 5 + 7 - 9 + 11 - 13 + 15 - 17$ .
14. Compute  $1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 + 19$ .
15. Compute  $1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} + \frac{1}{32}$ .
16. Compute  $\frac{1}{2} + \frac{1}{6} + \frac{1}{12} + \frac{1}{20} + \frac{1}{30} + \frac{1}{42}$ .
17. Compute  $\sqrt{15}\sqrt{21}\sqrt{35}$ .
18. Compute  $1^2 + 2^2 + 3^2 + \dots + 12^2$ .
19. Compute  $(-1)^0 + (-1)^1 + (-1)^2 + \dots + (-1)^{2016}$ .
20. Compute  $20^3 + 19^3 + \dots + 16^3$ .
21. What is the product of the greatest common divisor and least common multiple of 20 and 16?
22. Let  $S$  be the set of numbers obtained by permuting the digits of 123. What is the sum of the numbers in  $S$ ?

23. Lucas is 20 years old and Raymond is 16 years old. How many years ago was it when Lucas was twice as old as Raymond?
24. How many distinct positive prime factors does 15015 have?
25. How many (nondegenerate) triangles can be made using points from the grid below?



26. Suppose that you made a pie for Thanksgiving. Lynda ate  $\frac{1}{2}$  of the pie. Then Kenia ate  $\frac{1}{3}$  of what was left. What fraction of the pie do you have left to eat?
27. Find the area of the circle that circumscribes a triangle with side lengths 6, 8, and 10.
28. Alvin the Anteater lives on the number line and initially starts at 0. Every second, he moves one unit in either direction at random. What is the probability that after 13 moves, his location is positive?
29. What is the area of the largest circle that will fit inside a square with side length 12?
30. Anna is a cow tied to the corner of a barn with a rope of length 2. The barn is rectangular with width 1 and length 3. What is the total area of the region over which Anna can roam if she cannot go into the barn?
31. Alice only likes integers that are perfect squares. Bob only likes integers that leave a remainder of 1 when divided by 3. How many integers less than 1000 are liked by both Alice and Bob?
32. If  $x + y = 20$  and  $xy = 16$ , what is  $x^2 + y^2$ ?
33. What is the remainder when  $20^{16}$  is divided by 7?
34. 8 students are sitting around a circular table. If you randomly pick 3 of these students, what is the probability they are all seated next to each other?
35. Suppose that  $(x, y)$  is a solution to the equation  $x^2 + y^2 = 1$ . What is the maximum value of  $xy$ ?
36. A teacher is returning exams to 5 students. In how many ways can she return the exams such that no student gets their own exam?
37. Freddy rolls a 20-sided die (with sides labeled 1 through 20) 16 times, and sums the results. What is the most probable sum?
38. What is the coefficient of the  $x^2$  term in the polynomial  $(2x + 4)(3x^2 + 4)$ ?
39. How many real solutions are there for the equation  $\frac{x}{y} + \frac{y}{x} = 1$ ?
40. How many three digit numbers have digits whose product is 12?
41. Define the operation  $a \otimes b = a^2 + b^2$ . Find  $(1 \otimes 2) \otimes 3$ .

42. Let  $C$  be a circle of radius 21 with center  $O$ , and let  $A$  be a point 29 units away from  $O$ . Let  $B$  be a point on  $C$  such that  $\overrightarrow{AB}$  is tangent to  $C$ . Compute the area of triangle  $ABO$ .
43. Consider the line segment from  $(0, 0)$  to  $(320, 180)$ . How many points on this line segment have coordinates that are both integers? Include the endpoints  $(0, 0)$  and  $(320, 180)$ .
44. Define  $a \oplus b = a + ab$ . Compute  $(7 \oplus 8) \oplus 9 - 7 \oplus (8 \oplus 9)$ .
45. How many ways are there to choose two different students out of a classroom of 64 students?
46. If a rectangle has perimeter 2016, what is its maximum possible area?
47. How many proper divisors does 2016 have?
48. How many positive integers less than or equal to 2016 are multiples of 20 or 16, but not both?
49. Find the 2016th letter of the infinite sequence: *BERKELEYBERKELEY* . . . .
50. What year is it?