1. David is taking a 50-question test, and he needs to answer at least 70% of the questions correctly in order to pass the test. What is the minimum number of questions he must answer correctly in order to pass the test?

   Answer: 35

   Solution: We simply compute 70% of 50 is $(0.7)(50) = \boxed{35}$.

2. You decide to flip a coin some number of times, and record each of the results. You stop flipping the coin once you have recorded either 20 heads, or 16 tails. What is the maximum number of times that you could have flipped the coin?

   Answer: 35 balls

   Solution: The maximum number of times you can flip without hitting either 20 heads or 16 tails is 34 balls: by flipping 19 heads, and 15 tails. At this point, you must stop after the next coin you flip, meaning that you have hit the maximum number of flips. Therefore, the maximum number of times you can flip is 35.

3. The width of a rectangle is half of its length. Its area is 98 square meters. What is the length of the rectangle, in meters?

   Answer: 14

   Solution: Let the length be x. We can use it to set up the equation $\frac{x^2}{2} = 98$, for which the solution is $x = \boxed{14}$.

4. Carol is twice as old as her younger brother, and Carol's mother is 4 times as old as Carol is. The total age of all three of them is 55. How old is Carol's mother?

   Answer: 40

   Solution: If Carol's brother's age is x, then the equation that we need to solve is $x+2x+8x = 55$. We see that $x = 5$, and so the mother's age is $8x = \boxed{40}$.

5. What is the sum of all two-digit multiples of 9?

   Answer: 585

   Solution: The numbers to sum will be 2×9 through 11×9. This gives

$$9\left(\sum_{i=2}^{11} i\right) = 9 \times 65 = \boxed{585}.$$

6. The number 2016 is divisible by its last two digits, meaning that 2016 is divisible by 16. What is the smallest integer larger than 2016 that is also divisible by its last two digits?

   Answer: 2020

   Solution: The number 2000 is divisible by the last two digits. This is true for 16, and next for 20. The next integer after 2016 that is also divisible by its last two digits is $\boxed{2020}$.

7. Let Q and R both be squares whose perimeters add to 80. The area of Q to the area of R is in a ratio of $16 : 1$. Find the side length of Q.

   Answer: 16

   Solution: Let q and r be the side lengths of Q and R respectively. We can set up two equations: $4q + 4r = 80$ and $\frac{q^2}{r^2} = 16$. Solving these two equations gives $q = \boxed{16}$.

8. How many 8-digit positive integers have the property that the digits are strictly increasing from left to right? For instance, 12356789 is an example of such a number, while 12337889 is not.

   Answer: 9

   Solution: We note that all the digits of the number must be unique, and if our number included 0, it would have to be the leftmost digit. However, it would then be a leading zero, so our number cannot have zero. Our solution is then simply choosing 8 numbers out of 9 available digits, which is $\boxed{9}$.

9. During a game, Steve Korry attempts 20 free throws, making 16 of them. How many more free throws does he have to attempt to finish the game with 84% accuracy, assuming he makes them all?

   Answer: 5

   Solution: Note that $\frac{16+x}{20+x} = 0.84$. Solving the equation, we see that $x = \boxed{5}$.

10. How many different ways are there to arrange the letters $MILKTEA$ such that TEA is a contiguous substring?

    For reference, the term "contiguous substring" means that the letters TEA appear in that order, all next to one another. For example, $MITEALK$ would be such a string, while $TMIELKA$ would not be.

    Answer: 120

    Solution: We consider TEA as one letter, leaving 5 things to be permuted, with no indistinguishable letters. The answer is thus $5! = \boxed{120}$.

11. Suppose you roll two fair 20-sided dice. What is the probability that their sum is divisible by 10?

    Answer: $\mathbf{\frac{1}{10}}$

    Solution: No matter what the result of your first roll is, there are exactly 2 numbers, out of 20 possible, that you can roll on the second die which would give you a sum that is divisible by 10. Thus, $\boxed{\frac{1}{10}}$ is the probability of this event.

12. Suppose that two of the three sides of an acute triangle have lengths 20 and 16, respectively. How many possible integer values are there for the length of the third side?

    Answer: 13

    Solution: Suppose that the third side length is c. By the Pythagorean Theorem, we'll need $c^2 > 20^2 - 16^2 = 144$ and $c^2 < 20^2 + 16^2 = 656$. This means $12 < c \leq 25 < \sqrt{656} < 26$. Thus, c can be any integer value between 13 and 25, inclusive, which gives us $\boxed{13}$ possible values.

13. Suppose that between Beijing and Shanghai, an airplane travels 500 miles per hour, while a train travels at 300 miles per hour. You must leave for the airport 2 hours before your flight, and must leave for the train station 30 minutes before your train. Suppose that the two methods of transportation will take the same amount of time in total. What is the distance, in miles, between the two cities?

    Answer: 1125

    Solution: Suppose x is the number of hours that the flight took. The train essentially has a head start of 1.5 hours, or $1.5 \times 300 = 450$ miles, on the plane. Therefore, we can solve

$500x = 300x + 450$ for the amount of time that the plane took. The solution to this equation is $x = \frac{9}{4}$, so the distance must be $500x = \frac{500\times 9}{4} = 125 \times 9 = \boxed{1125}$ miles.

14. How many nondegenerate triangles (triangles where the three vertices are not collinear) with integer side lengths have a perimeter of 16? Two triangles are considered distinct if they are not congruent.

    Answer: 5

    Solution: We need to count the ways we can choose 3 integers that sum to 16 subject to the triangle inequality. We can split into cases based on the largest side length, denoted by ℓ.

    Case $\ell = 1, 2, 3, 4, 5$: The largest triangle can have perimeter 3ℓ, so clearly there are no solutions with $\ell < 6$.

    Case $\ell = 6$: We have solutions $(6, 6, 4)$ and $(6, 5, 5)$.

    Case $\ell = 7$: We have solutions $(7, 7, 2)$, $(7, 6, 3)$, and $(7, 5, 4)$.

    Case $\ell \geq 8$: We note that if $\ell \geq 8$, the other two sides must sum up to $16 - \ell \leq 8$, which would be less than ℓ and thus form a degenerate triangle.

    Our answer is thus $\boxed{5}$.

15. John can drive 100 miles per hour on a paved road and 30 miles per hour on a gravel road. If it takes John 100 minutes to drive a road that is 100 miles long, what fraction of the time does John spend on the paved road?

    Answer: $\frac{3}{7}$

    Solution: In order to solve this problem, we can set up the following system of 2 equations. The equation $x + y = \frac{5}{3}$ gives that the total travel time is 100 minutes, equal to $\frac{5}{3}$ hours. The equation $100x + 30y = 100$ gives that the total mileage of the roads is 100 kilometers. Solving this system of equations gives us $x = \frac{5}{7}$. The ratio of time that John spends on the paved road is therefore $\frac{\frac{5}{7}}{\frac{5}{3}} = \boxed{\frac{3}{7}}$.

16. Alice rolls one pair of 6-sided dice, and Bob rolls another pair of 6-sided dice. What is the probability that at least one of Alice's dice shows the same number as at least one of Bob's dice?

    Answer: $\frac{37}{72}$

    Solution: Suppose Alice has rolled her dice. There are two cases: either she rolled two different numbers, or two of the same number. The first case happens with probability $\frac{5}{6}$, and the other with probability $\frac{1}{6}$. In the first case, the only way that Bob doesn't get a matching die is if neither of his dice show either of her two numbers, which happens with probability $\frac{4\cdot 4}{36}$, so the probability that it does happen is $\frac{20}{36}$. Meanwhile, the probability that one of Bob's dice matches the number showing on Alice's dice if both are the same, goes down to $\frac{36-25}{36} = \frac{11}{36}$. Weighing these cases by the probabilities that they occur, our final answer is

$$\frac{5}{6} \cdot \frac{20}{36} + \frac{1}{6} \cdot \frac{11}{36} = \frac{111}{216} = \boxed{\frac{37}{72}}.$$

17. When 20^{16} is divided by 16^{20} and expressed in decimal form, what is the number of digits to the right of the decimal point? Trailing zeroes should not be included.

    Answer: 48

    Solution: We can factor 20^{16} as $5^{16} \times 4^{16}$. Then, $\frac{20^{16}}{16^{20}} = \frac{5^{16} \times 2^{32}}{2^{80}} = \frac{5^{16}}{2^{48}}$. Each factor of 2 in the denominator will result in one extra digit to the right of the decimal, and so we'll have $\boxed{48}$ digits there.

18. Suppose you have a 20×16 bar of chocolate squares. You want to break the bar into smaller chunks, so that after some sequence of breaks, no piece has an area of more than 5. What is the minimum possible number of times that you must break the bar?

    For an example of how breaking the chocolate works, suppose we have a 2×2 bar and wish to break it entirely into 1×1 bars. We can break it once to get two 2×1 bars. Then, we would have to break each of these individual bars in half in order to get all the bars to be size 1×1, and we end up using 3 breaks in total.

    Answer: 63

    Solution: First, we will prove that it is not possible to use fewer than 63 cuts. Notice that in order to satisfy the conditions, we will need at least $\frac{20 \times 16}{5} = 64$ pieces. Each cut will create exactly 1 more piece. Therefore, we need at least 63 cuts. In order to see that it can be done in 63 cuts, first, make 15 cuts, each one parallel with the side of length 20, so that we have 16 strips of length 20. Cutting each of these strips into 4 equal parts will give us only chunks of area 5, and will take 3 cuts per chunk. We make $15 + 3(16) = \boxed{63}$ cuts total in order to do this.

19. A class of 10 students decides to form two distinguishable committees, each with 3 students. In how many ways can they do this, if the two committees can have no more than one student in common?

    Answer: 11760

    Solution: First, suppose that there are no members in common. The number of ways to do the selection is $\binom{10}{3} \cdot \binom{7}{3} = 4200$. Now, suppose that they have 1 member in common. Any of 10 people could be the one in common, and then we have to select 2 people each for the committees. We can do this in $10 \cdot \binom{9}{2} \cdot \binom{7}{2} = 10(36)(21) = 7560$ ways. Adding these up, we get $4200 + 7560 = \boxed{11760}$.

20. You have been told that you are allowed to draw a convex polygon in the Cartesian plane, with the requirements that each of the vertices has integer coordinates whose values range from 0 to 10 inclusive, and that no pair of vertices can share the same x or y coordinate value (so for example, you could not use both $(1, 2)$ and $(1, 4)$ in your polygon, but $(1, 2)$ and $(2, 1)$ is fine). What is the largest possible area that your polygon can have?

    Answer: 82

    Solution: We can obtain such a polygon with area 82 by taking the vertices

    $$\{(1, 0), (0, 9), (9, 10), (10, 1)\},$$

    which is a square with side length $\sqrt{82}$. To see that we can do no better, consider the whole region, which is the square the following vertices:

    $$\{(0, 0), (0, 10), (10, 0), (10, 10)\}.$$

For each of its outer "strips" of width 1 (so, for example, $\{(0,0),(0,1),(10,0),(10,1)\}$), more than half that strip's area will lie outside any polygon that you can draw, which means that we cannot cover an area of at least 20. However, the corner unit squares of area 1 are double counted, but by our previous reasoning, we also cannot cover more than half of their area with any polygon, so we cannot cover an area of at least 18. Our solution leaves an area of exactly area 18 uncovered, so we must have found the best possible solution. This gives an optimal area of $\boxed{82}$.