

## 1 Multiple Choice

0. You have no choice but to have fun. Have a free point. Good luck!
1. How many choices does this question have? **(A)** 1
2. There are no True/False questions on this test. **(A)** True **(B)** False
3. Which is larger,  $2015^{2016}$  or  $2016^{2015}$ ? **(A)**  $2015^{2016}$  **(B)**  $2016^{2015}$  **(C)** They are equal.
4. In terms of getting points in this section, which guessing strategy would net the most expected points? (Disregard the actual answers to this test.) **(A)** Ignore this section and work on the Free Response. **(B)** Always choose the last choice. **(C)** Always choose C. **(D)** B and C perform equally well.
5. If you roll a fair 4-sided die (numbered 1 through 4) and a fair 6-sided die (numbered 1 through 6), what is the probability that the sum of the rolls is prime? **(A)**  $\frac{1}{3}$  **(B)**  $\frac{3}{8}$  **(C)**  $\frac{5}{12}$   
**(D)**  $\frac{11}{24}$  **(E)**  $\frac{1}{2}$
6. Order the following mathematicians' birthdays from earliest to most recent: Leonhard Euler, Pierre Fermat, Carl Gauss. (Denote their names as E, F, and G respectively.) **(A)** *EFG*  
**(B)** *EGF* **(C)** *FEG* **(D)** *FGE* **(E)** *GEF* **(F)** *GFE*
7. What day of the week will it be 2015 days from now? **(A)** Sunday **(B)** Monday **(C)** Tuesday  
**(D)** Wednesday **(E)** Thursday **(F)** Friday **(G)** Saturday
8. True/False Redux! Which of the following statements are true:
- I. There were Fields Medalists this year.  
 II. Someone has received \$1,000,000 for solving a Millenium Problem.  
 III. Fermat proved Fermat's Last Theorem.
- (A)** None of them are true **(B)** I only **(C)** II only **(D)** III only **(E)** I and II  
**(F)** I and III **(G)** II and III **(H)** All of them are true
9. Right triangle  $ABC$  with right angle  $\angle ABC$  has side lengths  $AB = 20$  and  $BC = 15$ . What is the shortest distance from point  $B$  to line  $AC$ ? **(A)** 8 **(B)** 9.6 **(C)** 10 **(D)**  $5\sqrt{5}$  **(E)** 12  
**(F)**  $5\sqrt{7}$  **(G)** 14.4 **(H)** 15 **(I)**  $5\sqrt{10}$
10. How many total choices (options to select, not total combinations of ways to select) are there in the multiple choice section? **(A)** 26 **(B)** 169 **(C)** 182 **(D)** 325 **(E)** 338 **(F)** 351 **(G)** 364  
**(H)** 625 **(I)** 676 **(J)** 26!
11. Suppose we have a drawer containing 2 blue socks, 3 red socks, 4 yellow socks, and 5 green socks. It is dark, so you cannot see what color socks you are taking. What is the fewest number of socks that you need to grab before you can be sure that you have 4 socks of the same color?  
**(A)** 4 **(B)** 5 **(C)** 6 **(D)** 7 **(E)** 8 **(F)** 9 **(G)** 10 **(H)** 11 **(I)** 12 **(J)** 13 **(K)** 14
12. If the mean of five positive integers is 40, what is the largest possible median? **(A)** 40 **(B)** 53  
**(C)** 66 **(D)** 67 **(E)** 78 **(F)** 79 **(G)** 80 **(H)** 98 **(I)** 99 **(J)** 100 **(K)** 196 **(L)** 200

13. What is the smallest positive integer to use each of the letters (I, V, X, L, C, D, M) when represented as a Roman numeral? (A) 1 (B) 556 (C) 666 (D) 941 (E) 1111 (F) 1234 (G) 1337 (H) 1444 (I) 1556 (J) 1664 (K) 1666 (L) 1987 (M) 2015
14. Given regular pentagon  $ABCDE$ , construct point  $F$  on  $BC$  such that  $\angle FAE = 90^\circ$ . What is the measure of  $\angle AFC$  in degrees? (A) 18 (B) 54 (C) 72 (D) 90 (E) 96 (F) 108 (G) 120 (H) 126 (I) 135 (J) 144 (K) 150 (L) 160 (M) 162 (N) 165
15. Suppose  $x + y = 20$  and  $xy = 15$ . What is  $x^3 + y^3$ ? (A) 370 (B) 385 (C) 400 (D) 415 (E) 430 (F) 625 (G) 1000 (H) 1625 (I) 2015 (J) 3375 (K) 6800 (L) 7100 (M) 7700 (N) 8300 (O) 8900
16. True/False Redux, Part 2! Which of the following statements are true:
- I.  $e^\pi - \pi = 20$
  - II. 8 and 9 are the only pair of perfect powers that differ by 1 (where a perfect power is defined to be  $a^b$  for integers  $a > 0, b > 1$ )
  - III. There are no positive integer solutions to  $x^3 + y^4 = z^5$ .
  - IV. The regular pentagon can be constructed with straightedge and compass.
- (A) None of them are true (B) I only (C) II only (D) III only (E) IV only (F) I and II (G) I and III (H) I and IV (I) II and III (J) II and IV (K) III and IV (L) All but I (M) All but II (N) All but III (O) All but IV (P) All of them are true
17. If there is a 30% chance of rain on Monday, and an 80% chance that the weather on Tuesday is the same as that of Monday, what is the probability of rain on Tuesday? (A) 11% (B) 14% (C) 20% (D) 24% (E) 26% (F) 28% (G) 30% (H) 35% (I) 38% (J) 42% (K) 44% (L) 46% (M) 49% (N) 50% (O) 56% (P) 70% (Q) 76%
18. A group of 17 people go to the movies. If it costs adults \$12 and children \$7, and the total cost is \$159, how many adults went? (A) 0 (B) 1 (C) 2 (D) 3 (E) 4 (F) 5 (G) 6 (H) 7 (I) 8 (J) 9 (K) 10 (L) 11 (M) 12 (N) 13 (O) 14 (P) 15 (Q) 16 (R) 17
19. Given triangle  $ABC$ , let the midpoint of  $BC$  be  $D$ . Triangle  $ABC$  is similar to triangle  $ADB$ . What is  $\frac{AC}{AB}$ ? (A)  $\frac{1}{2}$  (B)  $\frac{\sqrt{2}}{2}$  (C) 1 (D)  $\frac{\sqrt{5}}{2}$  (E)  $\frac{4}{3}$  (F)  $\sqrt{2}$  (G)  $\frac{3}{2}$  (H)  $\frac{5}{3}$  (I) 2 (J) 3 (K)  $\pi$  (L)  $\sqrt{10}$  (M) 4 (N)  $\sqrt{26}$  (O)  $2\pi$  (P) 9 (Q) 16 (R) 2015 (S) Cannot be determined with the information given
20. What is the sum of all the positive integers that divide into 2015? (A) 2015 (B) 2016 (C) 2021 (D) 2424 (E) 2442 (F) 2688 (G) 2880 (H) 3238 (I) 3279 (J) 3328 (K) 3540 (L) 3728 (M) 4030 (N) 4224 (O) 4242 (P) 4422 (Q) 4626 (R) 5028 (S) 5102 (T) 5120
21. How many triples of positive integers  $a, b, c$  are there such that  $a + 2b + 3c = 30$ ? (A) 9 (B) 10 (C) 15 (D) 17 (E) 18 (F) 21 (G) 23 (H) 26 (I) 29 (J) 31 (K) 33 (L) 34 (M) 35 (N) 42 (O) 43 (P) 44 (Q) 48 (R) 53 (S) 55 (T) 56 (U) 61

22. You have a bag of marbles with 20 white marbles and 15 black marbles. What is the probability that when you take 2 marbles out of the bag, one of them is white and the other is black?  
 (A) 0 (B)  $\frac{1}{35}$  (C)  $\frac{2}{35}$  (D)  $\frac{15}{119}$  (E)  $\frac{1}{7}$  (F)  $\frac{1}{5}$  (G)  $\frac{3}{14}$  (H)  $\frac{12}{49}$  (I)  $\frac{1}{4}$  (J)  $\frac{30}{119}$  (K)  $\frac{3}{11}$   
 (L)  $\frac{11}{35}$  (M)  $\frac{1}{3}$  (N)  $\frac{3}{8}$  (O)  $\frac{3}{7}$  (P)  $\frac{1}{2}$  (Q)  $\frac{60}{119}$  (R)  $\frac{4}{7}$  (S)  $\frac{9}{14}$  (T)  $\frac{3}{4}$  (U)  $\frac{15}{17}$  (V) 1
23. Compute the remainder if you divide  $2^{100}$  by 23. (A) 0 (B) 1 (C) 2 (D) 3 (E) 4 (F) 5  
 (G) 6 (H) 7 (I) 8 (J) 9 (K) 10 (L) 11 (M) 12 (N) 13 (O) 14 (P) 15 (Q) 16 (R) 17  
 (S) 18 (T) 19 (U) 20 (V) 21 (W) 22
24. Alice, Bob, Carol, and Dave are in a race; there were no ties. You know that no person ended up in the same place as their alphabetical order (so e.g. Alice didn't come in first). Also, the two females finished next to each other while the two males did not. (Alice and Carol are female; Bob and Dave are male.) In what order did they finish the race in, from first to last?  
 (A) ABCD (B) ABDC (C) ACBD (D) ACDB (E) ADBC (F) ADCB (G) BACD  
 (H) BADC (I) BCAD (J) BCDA (K) BDAC (L) BDCA (M) CABD (N) CADB  
 (O) CBAD (P) CBDA (Q) CDAB (R) CDBA (S) DABC (T) DACB (U) DBAC  
 (V) DBCA (W) DCAB (X) DCBA
25. Draw a  $5 \times 5$  square grid, then draw a circle of radius 5 centered at one of the corners of the grid. How many smaller squares does the circumference of the circle pass through? (If the circumference only touches the square at a corner, it doesn't count.) (A) 1 (B) 2 (C) 3 (D) 4 (E) 5  
 (F) 6 (G) 7 (H) 8 (I) 9 (J) 10 (K) 11 (L) 12 (M) 13 (N) 14 (O) 15 (P) 16 (Q) 17  
 (R) 18 (S) 19 (T) 20 (U) 21 (V) 22 (W) 23 (X) 24 (Y) 25
26. Given points  $A, B, C, D$  on a line (in that order), we have  $AB = 40$ ,  $BC = 18$ , and  $CD = 40$ . We construct two circles with diameters at  $AC$  and  $BD$ . These two circles intersect at points  $X, Y$ . What is the distance  $XY$ ? (A)  $\frac{20}{9}$  (B) 3 (C) 12 (D)  $4\sqrt{14}$  (E) 15 (F) 18 (G) 20  
 (H) 21 (I) 22 (J) 24 (K) 29 (L) 30 (M)  $2\sqrt{119}$  (N) 36 (O) 38 (P) 40 (Q) 42 (R) 44  
 (S) 48 (T) 49 (U) 50 (V) 78 (W) 98 (X) 720 (Y) 2015  
 (Z) Cannot be determined with the information given

## 2 Arithmetic

27. Compute  $(-2014) + (-2013) + (-2012) + \dots + 2014 + 2015 + 2016$ .
28. Compute  $111111111^2$ .
29. Compute  $2(10 + 0(10 + 1(10 + 5)))$ .
30. Compute  $7^8$ .

### 3 Algebra

31. In the song "Twelve Days of Christmas", you get many gifts over a period of 12 days. On day  $n$ , you get  $n + (n - 1) + (n - 2) + \dots + 1$  gifts to be specific. How many gifts do you get in total?
32. If it takes Xavier 5 hours to paint a wall, Yanni 4 hours, and Zelda 3 hours, how long will it take for the painters to collectively paint three walls?
33. Given that  $x + 4y = 2015$ , what is the maximum possible value of  $xy$ ?
34. If  $x^4 + x^{-4} = 194$ , what is  $x + \frac{1}{x}$ ?

### 4 Geometry

35. What is the largest angle in a quadrilateral whose angles form an arithmetic progression of ratio 2?
36. A rectangular prism has surface area of 162 and volume 126. If one side measures 6, what are the other two sides?
37. Square  $ABCD$  has side length 2015. Denote midpoints of sides  $AB$  and  $CD$  to be  $E$  and  $F$  respectively. What is the distance between lines  $CE$  and  $AF$ ?
38. A trapezoid has bases of lengths 21 and 49, and legs of lengths 25 and 17. What is the area of the trapezoid?

### 5 Counting/Probability

39. Marcia flips 99 fair coins and Naomi flips 100 fair coins. What is the probability that Naomi flips more tails than Marcia?
40. Roll three fair 6-sided dice. What is the probability that the product of the rolled numbers is prime?
41. There is a string of 10 Christmas lights in a row. Suddenly, 3 random lights burn out. What is the probability that no two burnt out lights are next to each other?
42. Dylan, Helen, Joseph, Kenny, and Mike are taking photos together in a straight line to celebrate their graduation. Mike wants stand next to Helen, and Dylan does not want to stand beside Kenny. Find the number of ways to take the photo.

### 6 Miscellaneous

43. If you guessed C on every question in the multiple choice, how many would you get right?
44. What if you guessed the last choice for each question?
45. How many times does 2015 appear printed in this test?
46. How many times is 2015 the answer to a question?

## 7 Mini Cyclic Relay

$x_{47}, x_{48}, x_{49}, x_{50}$  refer to the answers to those respective problems.

47. What is the area of a square with side length  $x_{50}$ ?
48. How many prime numbers are between 1 and  $x_{47}$ , inclusive?
49. Find the larger solution to the equation  $t^2 - 11t + x_{48} = 0$ .
50. What is the remainder when 2015 is divided by  $x_{49}$ ?