

## 1 True/False

1. Today is Saturday!
2. If Claire is born November 17 in the Year 1776, she is now 238 years old.
3. Every square is a rhombus.
4. There are finitely many odd prime numbers.
5. There are finitely many Fibonacci numbers that are perfect squares.
6. Any polyhedron can be decomposed into a bunch of tetrahedra.
7.  $(\sqrt{-1})(\sqrt{-1}) = \sqrt{(-1)(-1)}$
8. The first female recipient of a Fields medal was awarded the prize in 2014.
9. Over half of the seven Millennium Prize problems have been solved.
10. If you inscribe an equilateral triangle inside a circle, the triangle can cover over half the area of the circle.
11. One quarter of the numbers from 1 to 100 (inclusive) are prime.
12. The number of ways to obtain a sum divisible by 3 from rolling three standard six-sided dice is divisible by 3.
13. The tortoise and the hare are running a 5 mile race. The tortoise moves at a speed of 2 miles per hour, while the hare runs at a speed of 10 miles per hour. If the hare takes a 1.5 hour nap halfway through the race, the tortoise will win the race.

## 2 Arithmetic

14. What is  $20 \times 14$ ?
15. Find  $987 + 1234$ .

16. Compute  $1000011 + 1000001 - 1001100$ .
17. Compute  $1010101 \times 101$
18. What is the sum of the first five primes?
19. What is the sum of the first five factorials of positive integers?
20. What is  $1 - 7 - 21 - 35 - 35 - 21 - 7 + 1$ ?
21. Find  $1 - 2(3 + 4(5 - 6(7 + 8(9 - 10))))$
22. Compute  $13 \cdot 13 + 3(13 \cdot 7) + 7 \cdot 7$ .
23. Find  $n$  such that  $\sqrt{7 + \sqrt{-1 + \sqrt{16 + \sqrt{34 + n}}}} = 3$ .

### 3 Counting/Probability

24. What is the expected value of the sum of the values of 100 rolls of a die?
25. Compute the number of ways to, using only pennies, nickels, dimes, and/or quarters, make 25 cents.
26. If there are  $n$  animals, and half have two legs, half have four legs, then given a random leg, what is the probability it belongs to a four-legged animal?
27. Five circles are on a plane. What is the greatest number of intersections possible?
28. Let  $n$  be an integer greater than 3. What is the probability that in a random permutation of the first  $n$  positive integers, 1 is in between 2 and 3? For example, 2143 and 431562 are allowed, but 321 is not.
29. David wants to flip a coin three times and get 3 heads. However, since this is very unlikely, he has given himself the option to reflip any one of the three coin tosses. What is the probability that he does get 3 heads with at most one reflip?

30. How many ordered pairs  $(x, y)$  of nonnegative integers satisfy  $x + y \leq 12$ ?

## 4 Geometry

31. If a sphere has a volume of  $36\pi$ , what is the area of a circle with the same radius?
32. What is the area of a square whose sides are the same length as the sides of an equilateral triangle with area 4?
33.  $ABCDEFGH$  is a regular octagon with side length 2. What is the area of triangle  $\triangle ABE$ ?
34. Suppose Ralph is a rectangle with width-to-length ratio of  $\frac{1}{2}$ . If he is inscribed inside of Calvin, a circle, find the ratio of Calvin's circumference to Ralph's perimeter.
35. Coco wants to do origami, and she has a square of side length 1. However, she would prefer to use a regular octagon for her origami, so she decides to cut the four corners of the square to get a regular octagon. Once she does so, what will be the side length of the octagon Coco obtains?
36. In  $\triangle ABC$ ,  $AB = 6$ ,  $BC = 8$ , and there is a right angle at  $B$ . If  $M$  is the midpoint of side  $AC$  and  $D$  is the altitude from  $B$  to  $AC$ , what is the area of  $\triangle BMD$ ?
37.  $\triangle ABC$  has side lengths  $AB = 16$ ,  $BC = 30$ , and  $AC = 34$ . Point  $D$  is on  $BC$  such that  $BD = 26$  and point  $E$  is on  $AC$  such that  $CE = \frac{15}{2}$ . What is the area of  $\triangle DCE$ ?

## 5 Algebra

38. The polynomial  $x^4 + ax^3 + bx^2 + cx + d$  has a root at  $x = 0$  and a double root at  $x = -2$ . What is the value of  $d$ ?
39. Let  $x + 2y + 3z = 12083$ . What is  $x$  if  $x = y - 1 = z + 1$ ?

40. Let  $x$  be a real number that satisfies  $x^4 + 2x^3 + 4x^2 = 171$ ,  $2x^2 + x + 1 = 22$  and  $2x^3 + 3x = 63$ . Find the value of  $x$ .

41. A sequence is given by  $a_1 = 1$ ,  $a_2 = 2$  and for  $n \geq 2$ ,

$$a_n = \frac{a_{n-1} + 1}{a_{n-2}}.$$

What is  $a_{2014}$ ?

42. Find the sum of the squares of the roots of the polynomial  $x^2 + 2x - 1$ .

43. A book has a total of 2751 digits in all of its page numbers. How many pages does the book have?

44. There are 2014 boxes of socks, labeled 1 through 2014. In the  $n^{\text{th}}$  box, there are a total of  $n^2$  socks:  $n$  of them are white socks, and the rest of them are red socks. What fraction of the total number of socks in all the boxes is red?

## 6 Miscellaneous

45. What is the smallest positive integer that cannot be expressed as the difference of two prime numbers?

46. The positive integer  $n$  has the property that it is equal to 6 times the sum of its digits. What is  $n$ ?

47. An *unusual* number is a positive integer  $n$  whose largest prime factor is strictly greater than  $\sqrt{n}$ . What is the smallest four digit unusual number?

48. The answer to this question is the inverse of the square of the answer to the question below.

49. The answer to this question is the same as the answer to the question above.

50. How many problems in this Speed Round have 2014 as an answer?