

# Countdown Round

Berkeley mini Math Tournament

16 November 2013

# Problem 1

## Problem 1

You roll 3 dice. What is the probability that the product of the outcomes is a factor of 64?

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You roll 3 dice. What is the probability that the product of the outcomes is a factor of 64?

The answer is  $\frac{1}{8}$ .

## Problem 2

Suppose that points  $A, B, C, D$  lie on a circle in that order.

Segments  $AC$  and  $BD$  intersect at  $E$ . Given that  $AE = 1$ ,  $AD = 2$ , and  $BE = 3$ , find all possible values for the length of  $BC$ .

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The answer is 6.

# Problem 3

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Nine math students take a 100 point test. The average of their scores is 79. If a tenth student takes the test, what is the minimum score that she needs to get to raise the class average score to 81?

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The answer is 99.

# Problem 4

## Problem 4

Find the unit's digit of the expression  
 $2013^1 + 2013^2 + 2013^3 + \dots + 2013^{2013}$ .



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Find the unit's digit of the expression  
 $2013^1 + 2013^2 + 2013^3 + \dots + 2013^{2013}$ .

The answer is 3.

# Problem 5

## Problem 5

Find the integer closest to  $\pi^3 + \pi^2 + \pi + 1$ .

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The answer is 45.

# Problem 6

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Three circles with radii 1, 2, and 3 are mutually externally tangent. Find the area of the triangle formed by their centers.

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The answer is 6.

# Problem 7

## Problem 7

If  $A = (2, 6)$ ,  $P = (5, 15)$  and  $B = (7, 21)$ , what is the ratio of the lengths  $AP : PB$ ?

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The answer is 3 : 2.

# Problem 8

## Problem 8

How many three digit numbers are there with an odd number of odd digits?



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The answer is 450.

## Problem 9

A square and a triangle have equal area. Let  $P_s$  be the perimeter of the square, and  $P_t$  be the perimeter of the triangle. What is the maximum possible value of  $\frac{P_s^2}{P_t^2}$ ?

# Problem 9

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A square and a triangle have equal area. Let  $P_s$  be the perimeter of the square, and  $P_t$  be the perimeter of the triangle. What is the maximum possible value of  $\frac{P_s^2}{P_t^2}$ ?

The answer is  $\frac{4\sqrt{3}}{9}$ .

# Problem 10

## Problem 10

$m$  and  $n$  are positive integers satisfying  $m^2 + n^2 = 2000$ . What is the maximum possible value of  $mn$ ?

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The answer is 800.

# Problem 11

## Problem 11

You are given 9 coins of a special currency, of denominations 1 through 9. In how many ways can you distribute the coins into 5 stacks such that each stack has the same value? Assume the stacks are unordered.

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The answer is 1.

# Problem 12

## Problem 12

Every minute it is in your mouth, a stick of gum loses 20% of its current flavor. To the nearest minute, how long will the stick of gum have more than half its original flavor?



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The answer is 3.

# Problem 13

## Problem 13

Billfred painstakingly wrote all the odd numbers from 1 to 199 (inclusive) on the board. Fredbob accidentally erased the first  $n$  odd numbers. If the sum of the remaining numbers on the board is 9900, find  $n$ .

# Problem 13

## Problem 13

Billfred painstakingly wrote all the odd numbers from 1 to 199 (inclusive) on the board. Fredbob accidentally erased the first  $n$  odd numbers. If the sum of the remaining numbers on the board is 9900, find  $n$ .

The answer is 10.

# Problem 14

## Problem 14

An infinitely large army of soldiers lies along the line  $y = x + 2$  and advances in the positive  $x$ -direction at a constant speed of 1 unit per second. If Jacob is at  $(0, 0)$  and is running away from the soldiers at a constant speed of 2 units per second, locate where he will be in 2 seconds, assuming he takes the route distancing himself farthest from the nearest soldier.

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The answer is  $(2\sqrt{2}, -2\sqrt{2})$ .

# Problem 15

## Problem 15

What is the sum of all positive integers less than 105 that are relatively prime to 105?

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The answer is 2520.

# Problem 16

## Problem 16

In how many ways can you rearrange the letters in *ADJACENT* such that *T* is adjacent to at least one *A*?



# Problem 16

## Problem 16

In how many ways can you rearrange the letters in *ADJACENT* such that *T* is adjacent to at least one *A*?

The answer is 9360.

# Problem 17

## Problem 17

Suppose  $f(x) = x^2 + 28x - 2013$ .

What is the largest positive integer  $a$  such that  $f(a)$  is not positive?

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Suppose  $f(x) = x^2 + 28x - 2013$ .

What is the largest positive integer  $a$  such that  $f(a)$  is not positive?

The answer is 33.

# Problem 18

## Problem 18

Given triangle  $ABC$  with  $\overline{AB} = 6$ ,  $\overline{BC} = 8$ , and  $\angle ABC = 90^\circ$ . If  $BD$  is the altitude from  $B$  to  $AC$ , what is the length of  $AD$ ?

# Problem 18

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Given triangle  $ABC$  with  $\overline{AB} = 6$ ,  $\overline{BC} = 8$ , and  $\angle ABC = 90^\circ$ . If  $BD$  is the altitude from  $B$  to  $AC$ , what is the length of  $AD$ ?

The answer is  $\frac{18}{5}$ .

# Problem 19

## Problem 19

Three rectangles with integer side lengths are placed adjacent to each other to create a larger rectangle  $R$ . If the sum of the perimeters of the three rectangles is 42, what is the minimal possible area of  $R$ ?

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Three rectangles with integer side lengths are placed adjacent to each other to create a larger rectangle  $R$ . If the sum of the perimeters of the three rectangles is 42, what is the minimal possible area of  $R$ ?

The answer is 18.

## Problem 20

Each of the following self-referential logical statements has a truth value. Determine how many of them are true.

- ① This statement is both true and false.
- ② This statement is either true or false.
- ③ This statement is neither true nor false.
- ④ If this statement is true, then this statement is false.



## Problem 20

Each of the following self-referential logical statements has a truth value. Determine how many of them are true.

- ① This statement is both true and false.
- ② This statement is either true or false.
- ③ This statement is neither true nor false.
- ④ If this statement is true, then this statement is false.

The answer is 1.

# Problem 21

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How many non-congruent triangles with integer side lengths have perimeter 8?

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How many non-congruent triangles with integer side lengths have perimeter 8?

The answer is 1.

# Problem 22

## Problem 22

A two-digit decimal number  $XY_{10}$  is reversed to  $YX_{16}$  in hexadecimal. Find  $XY_{10}$ .

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A two-digit decimal number  $XY_{10}$  is reversed to  $YX_{16}$  in hexadecimal. Find  $XY_{10}$ .

The answer is 53.

# Problem 23

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Bob multiplies a number by 2 to get 100 as the answer. However, instead of multiplying he was supposed to add the 2. What is the correct answer?

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Bob multiplies a number by 2 to get 100 as the answer. However, instead of multiplying he was supposed to add the 2. What is the correct answer?

The answer is 52.

# Problem 24

## Problem 24

Given that the prime factorization of 123123123 is  $3^2 \cdot x \cdot y$ , with  $x < y$ , find  $y$ .



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Given that the prime factorization of 123123123 is  $3^2 \cdot x \cdot y$ , with  $x < y$ , find  $y$ .

The answer is 333667.

# Problem 25

## Problem 25

Two standard six-sided dice are rolled. If at least one of the dice has a 2 on top, what is the probability that the dice sum to 9?

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## Problem 25

Two standard six-sided dice are rolled. If at least one of the dice has a 2 on top, what is the probability that the dice sum to 9?

The answer is 0.

## Problem 26

100 Berkeley students are taking a math class. If 77 of them are math majors, 43 of them are statistics majors and 2 of them are computer science majors, what is the greatest number of students who are only music majors that could be in the class if no student is a triple major?

## Problem 26

100 Berkeley students are taking a math class. If 77 of them are math majors, 43 of them are statistics majors and 2 of them are computer science majors, what is the greatest number of students who are only music majors that could be in the class if no student is a triple major?

The answer is 23.

# Problem 27

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Evaluate  $47 \cdot 89$ .

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Evaluate  $47 \cdot 89$ .

The answer is 4183.

# Problem 28

## Problem 28

Two trains are approaching each other at 40 mph and 60 mph respectively. If they are currently 70 miles apart, in how many minutes will they meet?



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The answer is 42.